

Measurement of the strange spectral function in hadronic τ decays

The OPAL Collaboration

G. Abbiendi², C. Ainsley⁵, P.F. Åkesson^{3,y}, G. Alexander²², J. Allison¹⁶, P. Amaral⁹, G. Anagnostou¹, K.J. Anderson⁹, S. Arcelli², S. Asai²³, D. Axen²⁷, G. Azuelos^{18,a}, I. Bailey²⁶, E. Barberio^{8,p}, T. Barillari³², R.J. Barlow¹⁶, R.J. Batley⁵, P. Bechtel²⁵, T. Behnke²⁵, K.W. Bell²⁰, P.J. Bell¹, G. Bella²², A. Bellerive⁶, G. Benelli⁴, S. Bethke³², O. Biebel³¹, O. Boeriu¹⁰, P. Bock¹¹, M. Boutemeur³¹, S. Braibant⁸, L. Brigladori², R.M. Brown²⁰, K. Buesser²⁵, H.J. Burckhart⁸, S. Campana⁴, R.K. Carnegie⁶, A.A. Carter¹³, J.R. Carter⁵, C.Y. Chang¹⁷, D.G. Charlton¹, C. Ciocca², A. Csilling²⁹, M. Cuffiani², S. Dado²¹, A. De Roeck⁸, E.A. De Wolf^{8,s}, K. Desch²⁵, B. Dienes³⁰, M. Donkers⁶, J. Dubbert³¹, E. Duchovni²⁴, G. Duckeck³¹, I.P. Duerdoth¹⁶, E. Etzion²², F. Fabbri², L. Feld¹⁰, P. Ferrari⁸, F. Fiedler³¹, I. Fleck¹⁰, M. Ford⁵, A. Frey⁸, P. Gagnon¹², J.W. Gary⁴, G. Gaycken²⁵, C. Geich-Gimbel³, G. Giacomelli², P. Giacomelli², M. Giunta⁴, J. Goldberg²¹, E. Gross²⁴, J. Grunhaus²², M. Gruwe⁸, P.O. Günther³, A. Gupta⁹, C. Hajdu²⁹, M. Hamann²⁵, G.G. Hanson⁴, A. Harel²¹, M. Hauschild⁸, C.M. Hawkes¹, R. Hawkings⁸, R.J. Hemingway⁶, G. Herten¹⁰, R.D. Heuer²⁵, J.C. Hill⁵, K. Hoffman⁹, D. Horváth^{29,c}, P. Igo-Kemenes¹¹, K. Ishii²³, H. Jeremie¹⁸, P. Jovanovic¹, T.R. Junk^{6,i}, N. Kanaya²⁶, J. Kanzaki^{23,u}, D. Karlen²⁶, K. Kawagoe²³, T. Kawamoto²³, R.K. Keeler²⁶, R.G. Kellogg¹⁷, B.W. Kennedy²⁰, K. Klein^{11,t}, A. Klier²⁴, S. Kluth³², T. Kobayashi²³, M. Kobel³, S. Komamiya²³, T. Krämer²⁵, P. Krieger^{6,l}, J. von Krogh¹¹, K. Kruger⁸, T. Kuhl²⁵, M. Kupper²⁴, G.D. Lafferty¹⁶, H. Landsman²¹, D. Lanske¹⁴, J.G. Layter⁴, D. Lellouch²⁴, J. Letts^o, L. Levinson²⁴, J. Lillich¹⁰, S.L. Lloyd¹³, F.K. Loebinger¹⁶, J. Lu^{27,w}, A. Ludwig³, J. Ludwig¹⁰, W. Mader³, S. Marcellini², A.J. Martin¹³, G. Masetti², T. Mashimo²³, P. Mättig^m, J. McKenna²⁷, R.A. McPherson²⁶, F. Meijers⁸, W. Menges²⁵, S. Menke³², F.S. Merritt⁹, H. Mes^{6,a}, A. Michelini², S. Mihara²³, G. Mikenberg²⁴, D.J. Miller¹⁵, S. Moed²¹, W. Mohr¹⁰, T. Mori²³, A. Mutter¹⁰, K. Nagai¹³, I. Nakamura^{23,v}, H. Nanjo²³, H.A. Neal³³, R. Nisius³², S.W. O’Neale^{23,*}, A. Oh⁸, A. Okpara¹¹, M.J. Oreglia⁹, S. Orito^{23,*}, C. Pahl³², G. Pásztor^{4,g}, J.R. Pater¹⁶, J.E. Pilcher⁹, J. Pinfold²⁸, D.E. Plane⁸, B. Poli², O. Pooth¹⁴, M. Przybycień^{8,n}, A. Quadt³, K. Rabbertz^{8,r}, C. Rembser⁸, P. Renkel²⁴, J.M. Roney²⁶, S. Rosati^{3,y}, Y. Rozen²¹, K. Runge¹⁰, K. Sachs⁶, T. Saeki²³, E.K.G. Sarkisyan^{8,j}, A.D. Schaile³¹, O. Schaile³¹, P. Scharff-Hansen⁸, J. Schieck³², T. Schörner-Sadenius^{8,a1}, M. Schröder⁸, M. Schumacher³, W.G. Scott²⁰, R. Seuster^{14,f}, T.G. Shears^{8,h}, B.C. Shen⁴, P. Sherwood¹⁵, A. Skuja¹⁷, A.M. Smith⁸, R. Sobie²⁶, S. Söldner-Rembold¹⁵, F. Spano⁹, A. Stahl^{3,x}, D. Strom¹⁹, R. Ströhmer³¹, S. Tarem²¹, M. Tasevsky^{8,z}, R. Teuscher⁹, M.A. Thomson⁵, E. Torrence¹⁹, D. Toya²³, P. Tran⁴, I. Trigger⁸, Z. Trócsányi^{30,e}, E. Tsur²², M.F. Turner-Watson¹, I. Ueda²³, B. Ujvári^{30,e}, C.F. Vollmer³¹, P. Vannerem¹⁰, R. Vértési^{30,e}, M. Verzocchi¹⁷, H. Voss^{8,q}, J. Vossebeld^{8,h}, D. Waller⁶, C.P. Ward⁵, D.R. Ward⁵, P.M. Watkins¹, A.T. Watson¹, N.K. Watson¹, P.S. Wells⁸, T. Wengler⁸, N. Wermes³, D. Wetterling¹¹, G.W. Wilson^{16,k}, J.A. Wilson¹, G. Wolf²⁴, T.R. Wyatt¹⁶, S. Yamashita²³, D. Zer-Zion⁴, L. Zivkovic²⁴

¹ School of Physics and Astronomy, University of Birmingham, Birmingham B15 2TT, UK

² Dipartimento di Fisica dell’Università di Bologna and INFN, 40126 Bologna, Italy

³ Physikalisches Institut, Universität Bonn, 53115 Bonn, Germany

⁴ Department of Physics, University of California, Riverside CA 92521, USA

⁵ Cavendish Laboratory, Cambridge CB3 0HE, UK

⁶ Ottawa-Carleton Institute for Physics, Department of Physics, Carleton University, Ottawa, Ontario K1S 5B6, Canada

⁸ CERN, European Organisation for Nuclear Research, 1211 Geneva 23, Switzerland

⁹ Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago IL 60637, USA

¹⁰ Fakultät für Physik, Albert-Ludwigs-Universität Freiburg, 79104 Freiburg, Germany

¹¹ Physikalisches Institut, Universität Heidelberg, 69120 Heidelberg, Germany

¹² Indiana University, Department of Physics, Bloomington IN 47405, USA

¹³ Queen Mary and Westfield College, University of London, London E1 4NS, UK

¹⁴ Technische Hochschule Aachen, III Physikalisches Institut, Sommerfeldstrasse 26–28, 52056 Aachen, Germany

¹⁵ University College London, London WC1E 6BT, UK

¹⁶ Department of Physics, Schuster Laboratory, The University, Manchester M13 9PL, UK

¹⁷ Department of Physics, University of Maryland, College Park, MD 20742, USA

¹⁸ Laboratoire de Physique Nucléaire, Université de Montréal, Montréal, Québec H3C 3J7, Canada

¹⁹ University of Oregon, Department of Physics, Eugene OR 97403, USA

²⁰ CCLRC Rutherford Appleton Laboratory, Chilton, Didcot, Oxfordshire OX11 0QX, UK

²¹ Department of Physics, Technion-Israel Institute of Technology, Haifa 32000, Israel

²² Department of Physics and Astronomy, Tel Aviv University, Tel Aviv 69978, Israel

²³ International Centre for Elementary Particle Physics and Department of Physics, University of Tokyo, Tokyo 113-0033, and Kobe University, Kobe 657-8501, Japan

²⁴ Particle Physics Department, Weizmann Institute of Science, Rehovot 76100, Israel

²⁵ Universität Hamburg/DESY, Institut für Experimentalphysik, Notkestrasse 85, 22607 Hamburg, Germany

²⁶ University of Victoria, Department of Physics, P O Box 3055, Victoria BC V8W 3P6, Canada

²⁷ University of British Columbia, Department of Physics, Vancouver BC V6T 1Z1, Canada

²⁸ University of Alberta, Department of Physics, Edmonton AB T6G 2J1, Canada

²⁹ Research Institute for Particle and Nuclear Physics, 1525 Budapest, P O Box 49, Hungary

³⁰ Institute of Nuclear Research, 4001 Debrecen, P O Box 51, Hungary

³¹ Ludwig-Maximilians-Universität München, Sektion Physik, Am Coulombwall 1, 85748 Garching, Germany

³² Max-Planck-Institute für Physik, Föhringer Ring 6, 80805 München, Germany

³³ Yale University, Department of Physics, New Haven, CT 06520, USA

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Abstract. Tau lepton decays with open strangeness in the final state are measured with the OPAL detector at LEP to determine the strange hadronic spectral function of the τ lepton. The decays $\tau^- \rightarrow (K\pi)^-\nu_\tau$, $(K\pi\pi)^-\nu_\tau$ and $(K\pi\pi\pi)^-\nu_\tau$ with final states consisting of neutral and charged kaons and pions have been studied. The invariant mass distributions of 93.4% of these final states have been experimentally determined. Monte Carlo simulations have been used for the remaining 6.6% and for the strange final states including η mesons. The reconstructed strange final states, corrected for resolution effects and detection efficiencies, yield the strange spectral function of the τ lepton. The moments of the spectral function and the ratio of strange to non-strange moments, which are important input parameters for theoretical analyses, are determined. Furthermore, the branching fractions $B(\tau^- \rightarrow K^-\pi^0\nu_\tau) = (0.471 \pm 0.059_{\text{stat}} \pm 0.023_{\text{sys}}) \%$ and $B(\tau^- \rightarrow K^-\pi^+\pi^-\nu_\tau) = (0.415 \pm 0.053_{\text{stat}} \pm 0.040_{\text{sys}}) \%$ have been measured.

1 Introduction

The τ lepton is the only lepton heavy enough to decay into hadrons. A comparison of the inclusive hadronic decay rate of the τ lepton with QCD predictions allows the measurement of some fundamental parameters of the theory. The inputs to these studies are the spectral functions that measure the transition probability to create hadronic systems of invariant mass squared $s = m^2$.

The energy regime accessible in τ lepton decays can be divided into two different regions: the low energy regime which has a rich resonance structure where non-perturbative QCD dominates; and the high energy regime near the kinematic limit, $s = m_\tau^2 = (1.776 \text{ GeV})^2$, where perturbative QCD dominates at least for integrals over the spectral function of spectral weight type. The high energy regime in τ lepton decays provides an environment where the strong coupling constant α_s can be measured [1–6] because the perturbative expansion converges well and non-perturbative effects are small. The measurement of the non-strange spectral function of hadronic τ lepton decays [7–9] has provided one of the most accurate measurements of α_s , and some very stringent tests of perturbative QCD at relatively low mass scales [10]. The spectral function of strange decays allows additional and independent tests of QCD and a

^a and at TRIUMF, Vancouver, Canada V6T 2A3

^c and Institute of Nuclear Research, Debrecen, Hungary

^e and Department of Experimental Physics, University of Debrecen, Hungary

^f and MPI München

^g and Research Institute for Particle and Nuclear Physics, Budapest, Hungary

^h now at University of Liverpool, Dept of Physics, Liverpool L69 3BX, U.K.

ⁱ now at Dept. Physics, University of Illinois at Urbana-Champaign, USA

^j and Manchester University

^k now at University of Kansas, Dept of Physics and Astronomy, Lawrence, KS 66045, USA

^l now at University of Toronto, Dept of Physics, Toronto, Canada

^m current address Bergische Universität, Wuppertal, Germany

ⁿ now at University of Mining and Metallurgy, Cracow, Poland

^o now at University of California, San Diego, USA

^p now at Physics Dept Southern Methodist University, Dallas, TX 75275, USA

^q now at IPHE Université de Lausanne, 1015 Lausanne, Switzerland

^r now at IEKP Universität Karlsruhe, Germany

^s now at Universitaire Instelling Antwerpen, Physics Department, 2610 Antwerpen, Belgium

^t now at RWTH Aachen, Germany

^u and High Energy Accelerator Research Organisation (KEK), Tsukuba, Ibaraki, Japan

^v now at University of Pennsylvania, Philadelphia, Pennsylvania, USA

^w now at TRIUMF, Vancouver, Canada

^x now at DESY Zeuthen

^y now at CERN

^z now with University of Antwerp

^{a1} now at DESY

* Deceased

Table 1. Overview of all channels with net strangeness and their branching fractions [22]

Final State	$B_{\text{total}}/\%$	Measured in this Paper		Not Measured	
		τ decay	$B_{\text{PDG}}/\%$	τ decay	$B_{\text{PDG}}/\%$
$(\text{K})^- \nu_\tau$	0.686 ± 0.023			$\tau^- \rightarrow \text{K}^- \nu_\tau$	0.686 ± 0.023
$(\text{K}\eta)^- \nu_\tau$	0.027 ± 0.006			$\tau^- \rightarrow \text{K}^- \eta \nu_\tau$	0.027 ± 0.006
$(\text{K}\pi)^- \nu_\tau$	1.340 ± 0.050	$\tau^- \rightarrow \text{K}^- \pi^0 \nu_\tau$	0.450 ± 0.030		
		$\tau^- \rightarrow \bar{\text{K}}^0 \pi^- \nu_\tau$	0.890 ± 0.040		
$(\text{K}\pi\pi)^- \nu_\tau$	0.708 ± 0.068	$\tau^- \rightarrow \bar{\text{K}}^0 \pi^- \pi^0 \nu_\tau$	0.370 ± 0.040		
		$\tau^- \rightarrow \text{K}^- \pi^+ \pi^- \nu_\tau$	0.280 ± 0.050		
				$\tau^- \rightarrow \text{K}^- \pi^0 \pi^0 \nu_\tau$	0.058 ± 0.023
$(\text{K}^*(892)\eta)^- \nu_\tau$	0.029 ± 0.009			$\tau^- \rightarrow \text{K}^*(892)^- \eta \nu_\tau$	0.029 ± 0.009
$(\text{K}\pi\pi\pi)^- \nu_\tau$	0.150 ± 0.045	$\tau^- \rightarrow \text{K}^- \pi^+ \pi^- \pi^0 \nu_\tau$	0.064 ± 0.024		
				$\tau^- \rightarrow \bar{\text{K}}^0 \pi^- \pi^0 \pi^0 \nu_\tau$	0.026 ± 0.024
				$\tau^- \rightarrow \text{K}^- \pi^0 \pi^0 \pi^0 \nu_\tau$	0.037 ± 0.021
				$\tau^- \rightarrow \bar{\text{K}}^0 \pi^- \pi^+ \pi^- \nu_\tau$	0.023 ± 0.020
Sum	2.940 ± 0.099		2.054 ± 0.085		0.886 ± 0.051

measurement of the mass of the strange quark [1, 11–19]. An analysis based on ALEPH data has been published in [20, 21].

The strange spectral function of the τ lepton is obtained from the invariant mass spectra of the hadronic τ decay modes with net strangeness. The contributing channels are discussed in the following and they are summarized in Table 1. The only contribution to the one-meson final state¹ $\tau^- \rightarrow \text{K}^- \nu_\tau$ is not reconstructed in this analysis. The $(\text{K}\pi)^-$ final state has a total branching fraction of 1.340%. It consists of the decay modes $\tau^- \rightarrow \text{K}^- \pi^0 \nu_\tau$ ($B = 0.450\%$) and $\tau^- \rightarrow \bar{\text{K}}^0 \pi^- \nu_\tau$ ($B = 0.890\%$). The first one was fully reconstructed in this analysis. The latter consists of two decay chains: $\tau^- \rightarrow \text{K}_L^0 \pi^- \nu_\tau$ (50%) and $\tau^- \rightarrow \text{K}_S^0 \pi^- \nu_\tau$ (50%) where the K_S^0 decays to two charged (neutral) pions in roughly 68% (32%) of all cases. For all final states including neutral kaons, only those decay modes are considered where a K_S^0 decays into two charged pions. The modes not reconstructed are accounted for by efficiency corrections assuming isospin symmetry. The three-meson final state $(\text{K}\pi\pi)^-$ has a total branching fraction of $B = 0.708\%$. It consists of the decay modes $\bar{\text{K}}^0 \pi^- \pi^0 \nu_\tau$ ($B = 0.370\%$) and $\text{K}^- \pi^+ \pi^- \nu_\tau$ ($B = 0.280\%$) which were reconstructed in this analysis, and the decay mode $\text{K}^- \pi^0 \pi^0 \nu_\tau$ ($B = 0.058\%$). Finally, the four-meson final state $(\text{K}\pi\pi\pi)^-$ was detected via the decay $\tau^- \rightarrow \text{K}^- \pi^+ \pi^- \pi^0 \nu_\tau$ ($B = 0.064\%$). The remaining modes $\bar{\text{K}}^0 \pi^- \pi^0 \pi^0 \nu_\tau$ ($B = 0.026\%$), $\text{K}^- \pi^0 \pi^0 \pi^0 \nu_\tau$ ($B = 0.037\%$) and $\text{K}^- \pi^0 \pi^0 \pi^0 \nu_\tau$ ($B = 0.023\%$) were not reconstructed. Of all strange τ decay modes with one kaon and one or more pions in the final state, 93.4% have been reconstructed in this analysis. The remaining 6.6% and the final states including η mesons were taken from Monte Carlo simulation. Hence, 91.1% of all final states with masses above m_K that contribute to the strangeness spectral function have been measured in this analysis. This

¹ Throughout the text we refer only to the negatively charged τ lepton. Simultaneous treatment of the charge conjugate decay is always implied.

paper describes the selection of these dominant channels and the measurement of their invariant mass spectra using data collected with the OPAL detector during the LEP-I period from 1991 to 1995. The spectral function is then determined from these spectra and the spectral moments were calculated.

From an experimental point of view, one of the key issues of this analysis is the separation of charged kaons and pions via the measurement of energy loss in the OPAL jet chamber in the dense environment of multiprong τ lepton decays. Substantial improvements have been achieved compared to previous publications [23]. In particular, these improvements have made it possible to obtain a reliable dE/dx measurement in an environment where three tracks are very close to each other. The reconstruction of neutral pions is based on the study of shower profiles in the electromagnetic calorimeter. Furthermore the identification and reconstruction of τ lepton decays with $\text{K}_S^0 \rightarrow \pi^+ \pi^-$ has been achieved with high efficiency and good mass resolution.

The outline of this paper is as follows: Sect. 2 gives a short description of the OPAL detector concentrating on those components which are important for this analysis. In addition, the τ lepton selection and the Monte Carlo samples used are discussed. Section 3 continues with a discussion of the experimental aspects of this work. The selection of the strange hadronic τ lepton decays is described in Sect. 4. In Sect. 5, the results for the branching fractions, the strange spectral function and the spectral moments are presented together with a discussion of the systematic uncertainties. The results are summarized in Sect. 6.

2 Detector and data samples

2.1 The OPAL detector

A detailed description of the OPAL detector can be found elsewhere [25]. A short overview is given here of those

components that are vital for this analysis. Charged particles are tracked in the central detector, which is enclosed by a solenoidal magnet, providing an axial magnetic field of 0.435 T. A high-precision silicon micro-vertex detector surrounds the beam pipe. It covers the angular region of $|\cos\theta| \leq 0.8$ and provides tracking information in the $r-\phi$ direction² (and z from 1993) [26]. The silicon detector is surrounded by three drift chambers: a high-resolution vertex detector, a large-volume jet chamber and z -chambers.

The jet chamber measures the momentum and energy loss of charged particles over 98% of the solid angle. It is subdivided into 24 sectors in $r-\phi$, each containing a radial plane with 159 anode sense wires parallel to the beam pipe. Cathode wire planes form the boundaries between adjacent sectors. The 3D-coordinates of points along the trajectory of a track are determined from the sense wire position, the drift time ($r-\phi$) and a charge division measurement (z) on the sense wire. The combined momentum resolution of the OPAL tracking system is $\sigma_p/p^2 \approx 1.5 \cdot 10^{-3} \text{ GeV}^{-1}$. From the total charge on each anode wire, the energy loss dE/dx is calculated and used for particle identification. In the momentum range relevant for this analysis ($3 \text{ GeV} < p < 35 \text{ GeV}$) a separation between pions and kaons close to 2σ at the high end ($> 20 \text{ GeV}$) and above 2σ at the low end ($< 20 \text{ GeV}$) is achieved.

Outside the solenoid are scintillation counters which measure the time-of-flight from the interaction region and aid in the rejection of cosmic events. Next is the electromagnetic calorimeter (ECAL), which, in the barrel section, is composed of 9440 lead-glass blocks, approximately pointing to the interaction region, and covering the range $|\cos\theta| < 0.82$. Each block has a $(10 \times 10) \text{ cm}^2$ profile with a depth of 24.6 radiation lengths. The resolution of the ECAL in the barrel region, including the effects of the approximately one radiation length of material in front, is $\sigma_E/E = \sqrt{(0.16)^2 \text{ GeV}/E + (0.015)^2}$.

The hadron calorimeter (HCAL) is beyond the electromagnetic calorimeter and is instrumented with layers of limited streamer tubes in the iron of the solenoid magnet return yoke. The outside of the hadron calorimeter is surrounded by the muon chamber system, which is composed of four layers of drift chambers in the barrel region.

2.2 Selection of τ lepton candidates

For the selection of τ lepton candidates, the standard τ selection procedure described in [24] is used. The decay of the Z^0 produces a pair of back-to-back highly relativistic τ leptons. Their decay products are strongly collimated and well contained within cones of half-angle 35° . Therefore, each τ decay is treated separately. In order to have a precise and reliable dE/dx measurement and to avoid regions of non-uniform calorimeter response, this analysis is restricted to the region $|\cos\theta| < 0.68$. To reject background

from hadronic events, exactly two cones are required and a maximum of six good³ tracks in the event is allowed. This background is further reduced by requiring that the sum of the charges of all tracks in each individual cone is ± 1 and the net charge of the whole event is zero. A total of 162477 τ cone candidates survive these selection criteria with an estimated non- τ background fraction of 1.5%.

2.3 Simulation of events

The τ Monte Carlo samples used consist of 200 000 τ pair events generated at $\sqrt{s} = m_{Z^0}$ using KORALZ 4.02 [27] and a modified version of TAUOLA 2.4 [28]. Modifications were necessary because all four- and five-meson final states with kaons (signal as well as background) are missing in the standard version. Since the resonance structure of these channels is poorly known, only phase space distributions of these final states were generated. In addition, the resonance structure of various final states was modified to give a better description of the data [29,30]. The branching fractions of the decay channels with kaons are enhanced in this sample so that it comprises roughly a factor of ten more τ decays with kaons than expected from data. The Monte Carlo events are then reweighted to the latest branching fractions given in [22], which are used throughout the selection procedure. The Monte Carlo events were processed through the GEANT OPAL detector simulation [31].

The non- τ background was simulated using Monte Carlo samples that consist of 4 000 000 $q\bar{q}$ events generated with JETSET [32], 574 000 Bhabha events generated with BH-WIDE [33], 792 000 μ -pair events generated with KORALZ [27] and 1 755 000 two-photon events using PHOJET [34], F2GEN [35] and VERMASEREN [36,37].

3 Identification of hadrons in τ final states

3.1 Energy loss measurement in τ decays with three or more tracks

A crucial part of this analysis is the identification of charged kaons via energy loss measurement in the jet chamber. Since this is the only means of distinguishing between charged pions and kaons in OPAL, a very good understanding of the effects present in the multi-track environment in τ lepton decays is vital for any analysis that requires particle identification.

The high Lorentz boost ($\gamma \approx 25$) of the τ lepton results in its decay products being contained in a narrow cone with a typical opening angle of 5° . In those cases where the final state consists of more than one track, the dE/dx measurement is known to be no longer reliable [23]. A systematic shift in the dE/dx distribution is observed which leads to a misidentification of charged pions as kaons and

² In the OPAL coordinate system the x -axis points to the center of the LEP ring. The z -axis is in the e^- beam direction. The angle θ is defined relative to the z -axis and ϕ is the azimuthal angle with respect to the x -axis.

³ A good track has a minimum number of 20 hits in the jet chamber, a maximum $|d_0|$ of 2 cm, a maximum $|z_0|$ of 75 cm, at least 100 MeV transverse momentum and a maximum radius of the first measured point on the track of 75 cm.

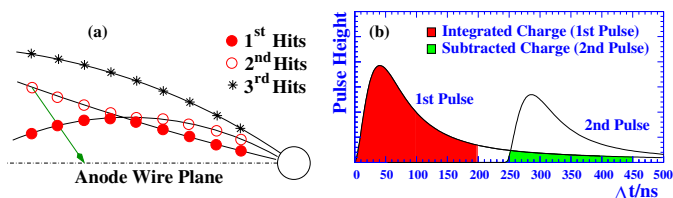


Fig. 1. Illustration of the tail subtraction procedure. **a** shows one half-sector of the OPAL central drift chamber with three tracks. The arrow indicates the drift direction of the ionization cloud following a path given by a Lorentz angle of 20° ; **b** illustrates the signal seen on the sense wire for two successive pulses

thus to a reduced sensitivity in those cases where particle identification is required. The reason is explained in the following text.

When a charged particle passes through the jet chamber, it produces an ionization cloud. This cloud travels at a constant speed of $v_D \approx 53 \mu\text{m/ns}$ in the homogeneous drift field to the sense wires, where it produces a signal pulse. The pulse is integrated over a time $\Delta t = 200 \text{ ns}$, which corresponds to a drift distance of $\sim 1 \text{ cm}$. The integral is proportional to the energy loss of the particle in this particular drift volume. The hits used for the energy loss measurement are the same as used in the track reconstruction, but they have to fulfill additional quality criteria [38]. In the following, they are called ‘ dE/dx hits’.

If an additional track passes through the same sector, a second pulse will be created which overlaps with the tail of the first one (Fig. 1). The contribution of the tail is determined by extrapolation of a reference pulse [38] which is normalized to the integral over the first pulse. The pulse used in all previous OPAL publications overestimates this tail contribution. The observed deviation in the energy loss measurement as a function of the distance between these two pulses (Δt) is shown in Fig. 2. This plot is obtained using tracks in τ data that have ‘first hits’ and ‘second hits’, and which have a momentum greater than 3 GeV . Such tracks arise when a second track traverses the same sector of the drift chamber and the two tracks have at least one intersection point in $r - \phi$ within the sensitive volume. Thus, for a given track, some of the hits do not need tail subtraction (‘first hits’) and some hits do need tail subtraction (‘second hits’). Using ‘first hits’ only (given there are at least 20 of them) $(dE/dx)_{\text{exp}}$ is calculated. The ‘second hits’ of the same track were then used to obtain $(dE/dx)_{\text{meas}}$ which is analyzed as a function of the drift time difference Δt between two successive hits. The left-right ambiguity, which in principle exists if two tracks pass on either side of the anode plane, is treated properly here, since the correction is applied based on measured drift times and not on the spatial separation of the tracks.

For two hits as close as $\Delta t = 200 \text{ ns}$, the observed deviation is of the order of 10% of the measured dE/dx . In the region between $400 \text{ ns} < \Delta t < 900 \text{ ns}$ the measured dE/dx is slightly overestimated. The standard correction is finally switched off for pulses with a drift time difference of more than 1000 ns which produces the structure shown in Fig. 2a. The deviation as observed for ‘third hits’ can be seen in Fig. 2b. When weighted with the Δt distribution of

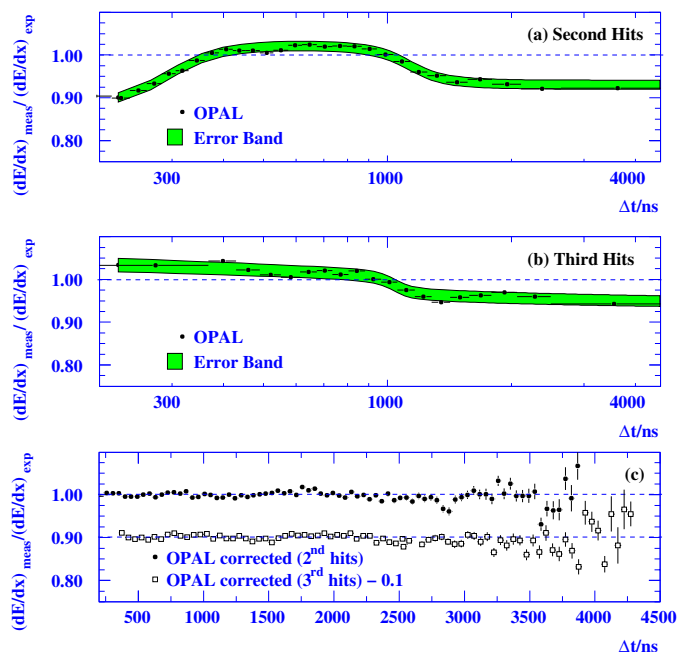


Fig. 2. Measured energy loss normalized to the expectation as a function of time difference between two measured hits ($200 \text{ ns} \cong 1.054 \text{ cm}$). The observed deviations using the standard correction are shown for **a** ‘second hits’ and **b** ‘third hits’. The error band reflects the 1σ error band of the parametrization. **c** shows the same distribution after all corrections have been applied. For better visibility, the distribution for ‘third hits’ is shifted by -0.1 in the plot

all measured hits this gives on average the correct dE/dx for high multiplicity events. In multiprong τ lepton decays this Δt distribution differs from the average in the sense that the distribution peaks at smaller Δt . Therefore, the energy loss measurement tends to be smaller than it should be. To avoid this problem, previous analyses [23] exploited the dE/dx information only of those tracks closest to the anode plane to classify the τ decay mode. This however significantly reduces the number of identified decays.

For this analysis, a new reference pulse has been developed that avoids the shortcomings of the standard one

$$P_{\text{norm}} = \left(p_1 \Delta t \exp\left(-\frac{\Delta t}{p_2}\right) + p_3 (\Delta t)^2 \exp\left(-\frac{(\Delta t)^2}{p_4}\right) \right) + \left(p_5 \Delta t \exp\left(-\frac{\Delta t}{p_6}\right) + p_7 (\Delta t)^2 \exp\left(-\frac{(\Delta t)^2}{p_8}\right) \right) + \tilde{p}_1 + \tilde{p}_2 \Delta t \quad (1)$$

with two terms to describe the short-range and the long-range part respectively plus a linear contribution. The p_i are parameters that are optimized for the multi-track environment in τ lepton decays. The correction is applied to all hits. The effect of the correction described above can be seen in Fig. 2c for ‘second hits’ and ‘third hits’. Apart from this normalization correction, a further bias reduction is obtained by also correcting the shape of the reference pulse depending on the charge deposited by the preceding pulse. In general, the new reference pulse shows a steeper rise

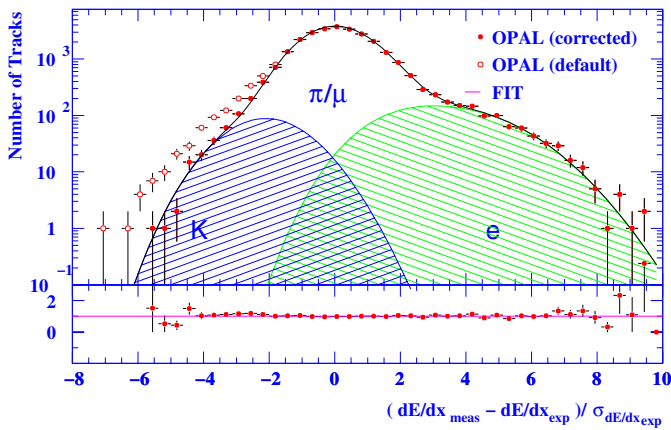


Fig. 3. Pull distribution obtained under a pion hypothesis for all tracks in 3-prong τ lepton decays with a minimum momentum of 3 GeV and a minimum number of 20 hits in the dE/dx measurement. Only the tracks with the same charge as the decaying τ lepton are shown. The solid points with error bars are data after all corrections and the function shows the expectation as explained in the text. The open points in the range between -8 to -2 show the same distribution but without the corrections mentioned in the text. The smaller plot at the bottom shows the ratio of the full data points to the sum of the functions

at low Δt and a lower tail to avoid overestimation of the tail subtraction for subsequent pulses as explained above. As a result of this procedure, a dE/dx bias reduction to $\pm 1\%$ in $(dE/dx)_{\text{meas}}/(dE/dx)_{\text{exp}}$ has been achieved. As the procedure is applied iteratively, i.e. the first pulse is used to correct the second, the first and corrected second pulse are used to correct a possible third pulse and so on, the chosen reference pulse is valid for any jet topology [38].

If a track is close to the anode or cathode plane in the jet chamber, the drift field is no longer homogeneous and one observes a deviation in the measured dE/dx of the order of 3%. Corrections for this effect are determined using $Z^0 \rightarrow \mu^- \mu^+$ events.

When a signal is measured at a sense wire, an induced signal at the neighboring wires is also present. This effect depends on the momentum (or curvature) of the track and on $\cos\theta$. Corrections have been determined using μ -pairs and $\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau$ decays. The effect is largest ($\sim 5\%$) for $\cos\theta \sim 0$ and large track momenta.

Finally, hits are discarded from tracks where the corresponding hit from the following or the preceding track is missing. In those cases, the measured dE/dx is overestimated since the charge is not correctly distributed among the hits but is assigned to one hit only. By discarding this kind of hits, (3–5)% of all dE/dx hits are lost.

The effect of all corrections can be seen in Fig. 3. It shows the dE/dx pull distribution under a pion-hypothesis for all like-sign⁴ tracks from 3-prong tau decays. The solid line is obtained by a fit of three Gaussians to the measured pull distribution in 9 variable momentum bins from 3 GeV to 40 GeV. Since the number of muons in this sample is very small and their energy loss is very similar to the one

for pions in the momentum range considered here, separate Gaussians for pions and muons are not needed. The free parameters in this fit are two of the three normalizations (the third one is constrained so that the sum of all three is equal to the observed number of tracks), one mean to allow for an overall shift (the position of the other two is calculated using the corresponding prediction from the Bethe-Bloch formula) and the width of the Gaussian which describes the contribution from pions. The deviation of this parameter from unity is used to obtain correction factors for the error of the energy loss measurement. The width of the two other Gaussians is calculated assuming that the relative error is constant.

From the measured energy loss, its error and the expectation calculated using the Bethe-Bloch equation, χ^2 probabilities are calculated that the measured energy deposition is in accordance with the expectation for a given particle type. Pion- and kaon-weights, W_π and W_K , as used in this paper, are then calculated by taking one minus the value of this probability. These weights acquire a sign depending on whether the actual energy loss lies above or below the expectation for a certain particle hypothesis. This means that W_π is expected to be close to -1 for kaons since their energy loss per unit length is smaller in the momentum range relevant in this analysis. For electron tracks, W_π is expected to be close to $+1$ due to the higher energy loss in this case. Whenever these quantities are used in the selection, a cut on at least 20 dE/dx hits for this track is made implicitly.

3.2 Photon reconstruction and identification of neutral pions

The reconstruction of π^0 mesons from photon candidates starts from an algorithm that has been used in previous OPAL publications (see e.g. [7]). It is based on the analysis of shower profiles in the ECAL as a function of the energy and direction of photons. In the fit each cluster is treated individually. Since the lead glass blocks in the barrel have a quasi-pointing⁵ geometry, only the lateral shower profile is considered. This algorithm provides the number of photon candidates and their corresponding 3-vectors that best describe the measured profile of the electromagnetic cluster. All photon candidates are assumed to originate from the interaction point (approximated by the primary vertex). Since the relative branching fractions of decays with and without π^0 mesons are different in strange and in non-strange final states, the parameters of the algorithm were optimized for this analysis as described below.

Within each cluster only blocks with an energy of at least 150 MeV are considered and the total cluster energy must exceed 600 MeV. The expected mean energy deposited by a minimum ionizing particle is then subtracted from all ECAL blocks hit by a charged particle before the fit. If a

⁵ The longitudinal axis is pointing towards the interaction region. The blocks are tilted slightly from a perfectly pointing geometry to prevent neutral particles from escaping through the gaps between blocks.

⁴ Tracks with the same charge as the initial τ lepton.

photon candidate is too close to the entrance point of a track into the ECAL, the track's hadronic interaction can distort the photon energy measurement. Therefore, a minimum angle of 2.8° between a photon candidate and a track's ECAL entrance point is required. This value is obtained from studies of the rate of fake π^0 mesons in the decay $\tau^- \rightarrow K^- \nu_\tau$ and subsequent optimization. To improve the energy resolution and the purity of the selection, a pairing algorithm is applied to recombine fake photon candidates with the closest photon candidates that were wrongly split up by the reconstruction procedure. The recombination is performed using the Jade jet finding scheme with the P0 option [39]. The jet resolution parameter y_{cut} values are optimized to obtain the best description of the number of expected photons using Monte Carlo events. Two y_{cut} values are determined, one for clusters where a track is pointing to one of the blocks in the cluster and one for those without tracks. This is necessary since the hadronic interaction of charged particles disturbs the shower profile. The optimized y_{cut} values are -3 and -4.6 for clusters with and without tracks, respectively. The angular resolution of this algorithm is 2.1° for clusters with tracks and 1.7° for clusters without tracks. Since these opening angles correspond directly to the energy of the neutral pion, photon candidates with an energy of more than 7.5 GeV are directly interpreted as neutral pions and the 4-vector is corrected to account for the π^0 mass. The energy of the reconstructed π^0 candidates for events with one and three tracks is shown in Fig. 4. The plots are normalized to the number of τ decays in the event sample. Neutral pion can-

didates with an energy below 1.5 GeV in the 1-prong case and 2 GeV in the 3-prong case are rejected.

For the remaining photon candidates, all two-photon combinations are tested. The combination which results in the maximum number of neutral pions with invariant two-photon masses not exceeding the π^0 mass by more than 1.5σ is retained. A fit with a π^0 mass constraint is then applied to all π^0 candidates. All π^0 candidates are assumed to originate from the primary interaction point.

3.3 Identification of K_S^0

The K^0 signal consists of 50% K_L^0 and 50% K_S^0 . The signature of a K_L^0 decay is a large energy deposit in the hadron calorimeter without an associated track pointing to the cluster. The resolution of the OPAL hadron calorimeter would not allow for a clean reconstruction of this channel, thus it is not considered here. For the K_S^0 , two decay modes are dominant, $K_S^0 \rightarrow \pi^0 \pi^0$ ($\approx 32\%$) and $K_S^0 \rightarrow \pi^+ \pi^-$ ($\approx 68\%$). In this analysis only the latter is considered since the photon reconstruction algorithm exploits the quasi-pointing geometry of the electromagnetic calorimeter in the barrel (see Sect. 3.2). Thus only photons from the primary vertex can be properly reconstructed.

The selection starts by combining each pair of oppositely charged tracks. Each track must have a transverse momentum with respect to the beam axis of $p_T \geq 150$ MeV, a minimum of 20 out of 159 possible hits in CJ, at least 20% of all geometrically possible hits and a maximum χ^2 for the track fit of 50. For each combination of tracks, their intersection points in the plane perpendicular to the beam axis are calculated. The one with a radius less than 150 cm is selected as the secondary vertex. If two vertices are found that satisfy this condition, the one with the first measured hit closest to the intersection point is selected. In addition, the z -coordinate of the vertex has to satisfy $|z_V| < 80$ cm.

The variables used in the selection described below are shown in Fig. 5. For each K_S^0 candidate, the angle between the reconstructed momentum of the candidate vertex and the K_S^0 direction of flight must be less than $|\Theta_V| < 0.5^\circ$. If the number of dE/dx -hits is more than 20 on at least one of the tracks, the energy loss information is used to veto photon conversions. All candidates where at least one track has a pion weight of more than 0.98 are rejected. To further reduce the background from photon conversions, the invariant mass of the K_S^0 candidate is calculated using electron hypothesis for both tracks. All combinations with a mass $m_{ee} < 0.1$ GeV are rejected. The remaining background that mainly comes from 3-prong τ lepton decays or from wrong track combinations is rejected by applying cuts on the impact parameters (with respect to the nominal interaction point d_0) of the two tracks. The d_0 values must have opposite sign according to the OPAL convention and the absolute values have to satisfy $\min(|d_0^{(1)}|, |d_0^{(2)}|) > 0.03$ cm and $\max(|d_0^{(1)}|, |d_0^{(2)}|) > 0.1$ cm.

The remaining K_S^0 candidates must have a momentum of $p_{K_S^0} > 3$ GeV. A 3D vertex fit is applied to each candidate, that includes a constraint of the invariant two-track mass

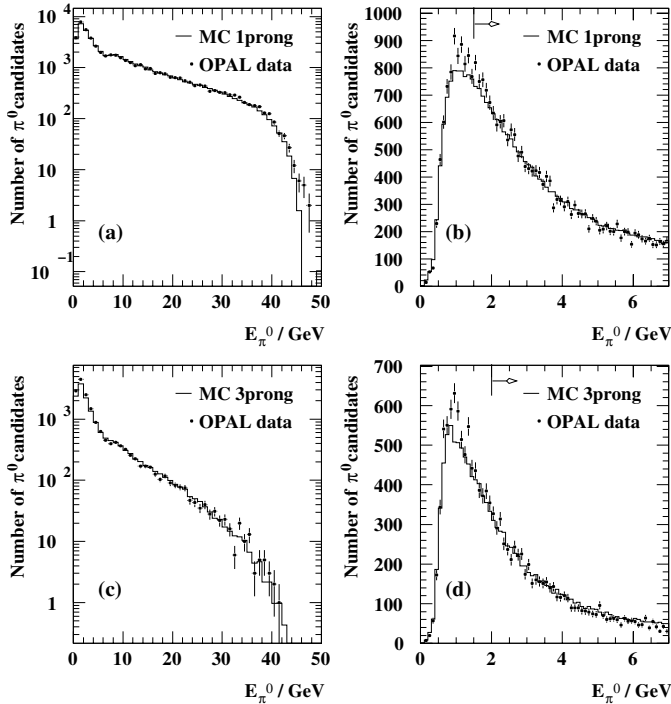


Fig. 4. Energy distribution of the reconstructed π^0 candidates. Plots **a** and **b** show the reconstructed energy for events with one track, plots **c** and **d** for events with three tracks. All plots are normalized to the number of τ lepton decays. The arrows indicate the region selected

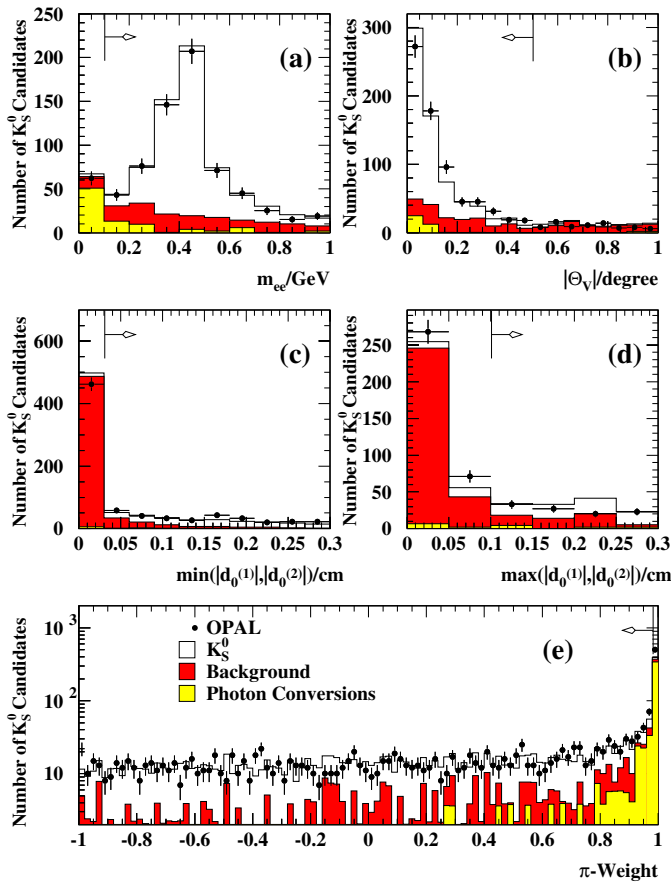


Fig. 5. Variables used in the K_S^0 selection. A detailed description of all variables is given in the text. The dots represent the data and the open histogram is Monte Carlo signal. The shaded areas show the background where photon conversions are marked separately. The arrows indicate the region selected. For all plots, all selection cuts have been applied except for the cut on the variable shown. All plots are normalized to the number of τ decays

under the pion hypothesis to the nominal K_S^0 mass. This is a 2C fit and a cut on the χ^2 probability at 10^{-5} is applied. The invariant two-track mass under the pion hypothesis before the kinematic fit can be found in Fig. 6 together with the radius of the reconstructed secondary vertex and the χ^2 probability of the kinematic fit. In Fig. 6a the $\pi\pi$ -invariant mass spectrum is shown without any vertex constraint. If more than one K_S^0 candidate shares the same track, the one with the smallest deviation from the nominal K_S^0 mass before the fit is selected.

After this selection procedure, a total of 535 K_S^0 candidates remain with an estimated purity of 82%. About 70% of the background consists of wrong combinations of tracks, and 30% comes from photon conversions. In one data event, two K_S^0 candidates are found within one cone. This event is considered to be background.

4 Identification of τ final states

For the selection of the various final states, a cut-based procedure is used where each τ decay is treated independently.

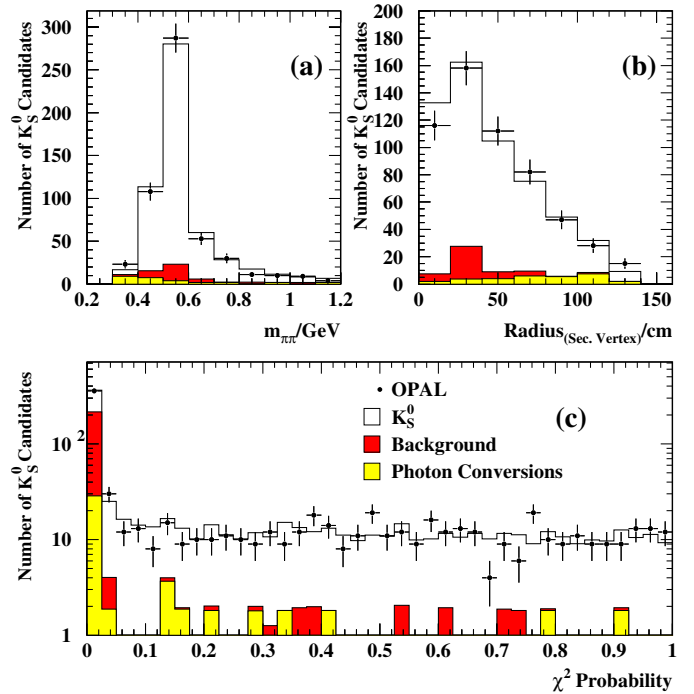


Fig. 6. Result of the K_S^0 selection. Plot a shows the invariant mass distribution of the K_S^0 candidates under pion hypothesis before the kinematic fit. Plot b shows the radius of the reconstructed secondary vertex and plot c the distribution of the χ^2 -probability of the 2C-fit. A cut is applied on the probability at 10^{-5} . The dots represent the data and the open histogram is the Monte Carlo signal. The shaded areas show the background where photon conversions are marked separately. All plots are normalized to the number of τ decays

For all selected decay modes, the cone axis, calculated from the momenta of all tracks and neutral clusters identified in the electromagnetic calorimeter, must have a polar angle within $|\cos\theta| < 0.68$ for the reasons explained above. Each selected cone must have at least one good track coming from the interaction point and the summed momenta of all tracks have to be less than the beam energy. Since there is at least one hadron in the final states considered here, the total energy deposited in the hadron calorimeter within the cone is required to exceed 1 GeV.

4.1 Corrections to the invariant mass spectra

The spectral function from τ lepton decays is a weighted invariant mass distribution of all hadronic final states with strangeness. The various final states have different experimental resolutions and migration effects that must be corrected for. To correct the observed data, the following method was used. The elements c_{ij} of the inverse detector response matrix were determined directly from Monte Carlo. They represent the probability that an event reconstructed in bin j was generated in bin i . To calculate this probability Monte Carlo samples for the signal channels with mass distributions according to phase space were used. The corrected distribution was then obtained by

$$g_i^{\text{Corrected}} = \sum_j c_{ij} \left(g_j^{\text{DATA}} - g_j^{\text{Background}} \right) \quad (2)$$

where g_j^{DATA} is the number of events in the data with a reconstructed mass in bin j , $g_j^{\text{Background}}$ is the number of background events predicted in bin j and $g_i^{\text{Corrected}}$ the number of events in mass bin i after correction.

The corrected distributions will in general be biased towards the Monte Carlo input distributions. To reduce the bias from this approach, the method was applied iteratively. The result of the preceding iteration was used to refine the elements of the inverse detector matrix. The optimal number of iterations was determined, using Monte Carlo simulations, to be two for all final states considered here. The corresponding systematic uncertainties are discussed in Sect. 5.4. Finally, detection efficiency corrections were applied to the corrected invariant mass distributions. The bin width of 150 MeV of the measured invariant mass spectra were chosen according to the mass resolution of the τ decay channels measured.

4.2 The $(K\pi)^-\nu_\tau$ final state

The $(K\pi)^-$ mass spectrum consists of two measured modes, $K^-\pi^0\nu_\tau$ and $\bar{K}^0\pi^-\nu_\tau$. From the latter decay mode, only the decays $K_S^0 \rightarrow \pi^+\pi^-$ are measured. The $K^*(892)$ dominance is well established in both cases.

4.2.1 $K^-\pi^0\nu_\tau$

In the $K^-\pi^0\nu_\tau$ selection, exactly one good track coming from the primary vertex is required. This track must have a minimum momentum of $p > 3$ GeV. For the track to be selected as a kaon, the pion weight has to satisfy $W_\pi < -0.98$ and the kaon weight $W_K < 0.6$. Furthermore, exactly one identified neutral pion is required with $E_{\text{min}}^{\pi^0} > 1.5$ GeV selected using the algorithm explained in Sect. 3.2. The variables used in this selection are shown in Figs. 7 and 8.

From this selection 360 events are seen in the data with 190.5 background events predicted from Monte Carlo.

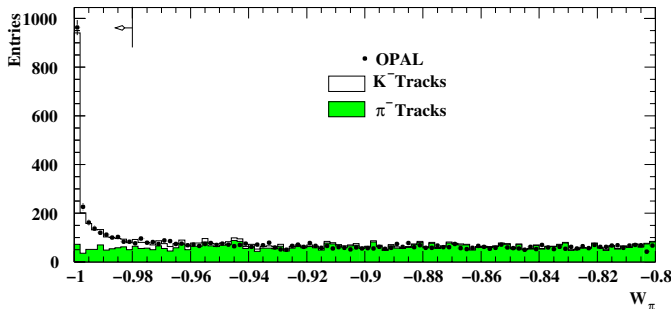


Fig. 7. The pion weight W_π for 1-prong τ decays in the range $(-1 : -0.8)$. The dots are the data points. The open (shaded) histogram denotes the contribution from kaon (pion) tracks as predicted by the Monte Carlo simulation. The plot is normalized to the number of τ decays. Events on the side of the direction of the arrow are considered kaon candidates

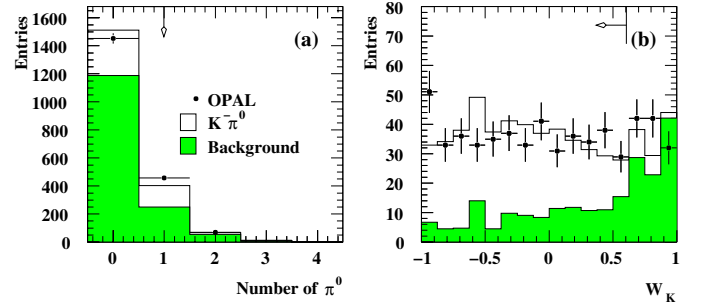


Fig. 8. Variables used in the $K^-\pi^0\nu_\tau$ selection. The dots are the data points and the open histogram is the prediction from the Monte Carlo. Plot **a** shows the number of reconstructed π^0 mesons with $E > 1.5$ GeV. In this plot the shaded area is the background prediction from the Monte Carlo. Plot **b** shows the kaon weight as explained in the text. Here the shaded area represents the expected background from pion tracks. The arrows indicate the events kept in the selection. For both plots, all selection cuts have been applied except for the cut on the variable shown

This corresponds to a background fraction of 54%. The invariant mass spectrum can be seen in Fig. 9. The mass resolution in this channel is approximately 40 MeV. The main background comes from $\tau^- \rightarrow K^-\nu_\tau$ decays where one fake neutral pion was reconstructed. Additional sources of background are $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$, $\tau^- \rightarrow K^-\bar{K}^0\nu_\tau$ and $\tau^- \rightarrow K^0K^-\pi^0\nu_\tau$ where the K^0 is a K_S^0 decaying to two

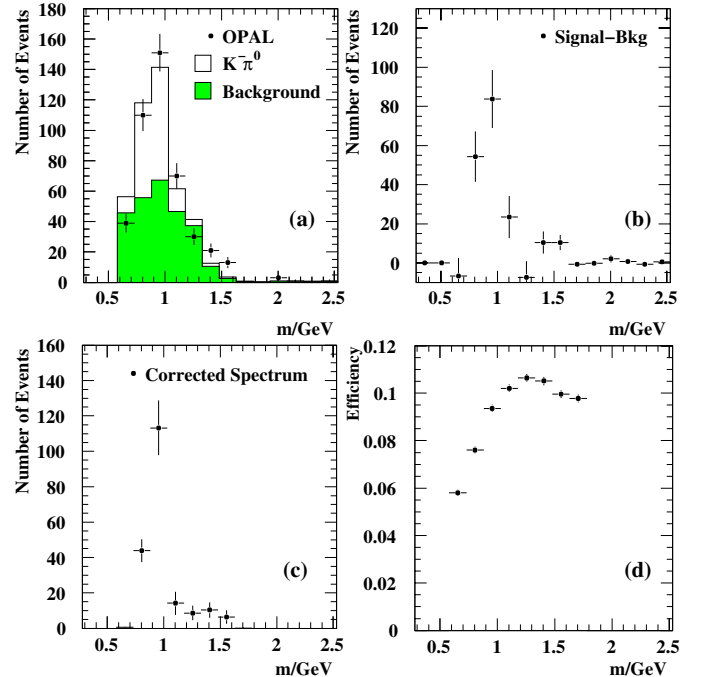


Fig. 9. Result of the $K^-\pi^0\nu_\tau$ selection. Plot **a** shows the measured invariant mass spectrum. The dots are the data, the open histogram is the Monte Carlo signal and the shaded area is the background. Plot **b** shows the background subtracted spectrum, **c** shows the corrected spectrum. Note that the correlation between neighboring bins in the corrected spectrum is of the order of 50%. Plot **d** is the selection efficiency as a function of the invariant mass

neutral pions or a K_L^0 which does not decay within the jet chamber.

4.2.2 $\bar{K}^0 \pi^- \nu_\tau$

The selection is very similar to that for the $K^- \pi^0 \nu_\tau$ final state. Here exactly one identified K_S^0 is required using the procedure from Sect. 3.3. In addition, one good track from the primary vertex is required. If the momentum of this track lies above the kinematically allowed minimum for a kaon, the same procedure as described above is used in order to identify $\tau^- \rightarrow K^- K^0 \nu_\tau$ decays which are a possible background source to this signal channel. Only events with zero reconstructed π^0 mesons are accepted. The variables used in the selection are shown in Fig. 10.

From this selection 361 events are expected with a background fraction of 47%, and 344 are seen in the data. The main background contributions come from decays $\tau^- \rightarrow K^0 \bar{K}^0 \pi^- \nu_\tau$, $\tau^- \rightarrow K^- K^0 \nu_\tau$ and $\tau^- \rightarrow \bar{K}^0 \pi^- \pi^0 \nu_\tau$ where the neutral pion escapes detection. The invariant mass spectrum for this channel is shown in Fig. 11. The mass resolution in this channel is approximately 60 MeV.

4.3 The $(K\pi\pi)^- \nu_\tau$ final state

The $(K\pi\pi)^-$ final state consists of the decay modes: $K^- \pi^+ \pi^- \nu_\tau$, $\bar{K}^0 \pi^- \pi^0 \nu_\tau$ and $K^- \pi^0 \pi^0 \nu_\tau$. In this analysis, the first two are reconstructed from data, the last one is taken from Monte Carlo simulation. To select the two former final states, the following procedure is applied.

4.3.1 $K^- \pi^+ \pi^- \nu_\tau$

The selection starts by requiring exactly three good tracks coming from the interaction point. These tracks are fitted to a common vertex and the fit probability is required to be larger than 10^{-7} . In addition, each pair of oppositely charged tracks has to fail the selection criteria for neutral

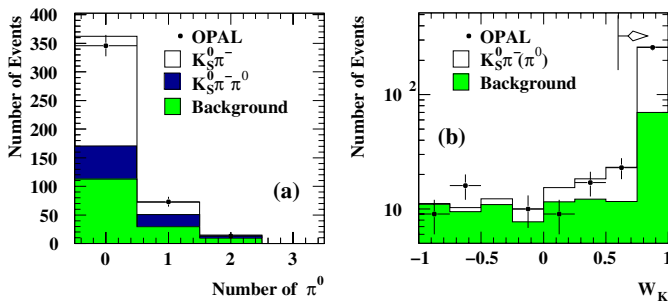


Fig. 10. Variables used in the $\bar{K}^0 \pi^- \nu_\tau$ and $\bar{K}^0 \pi^- \pi^0 \nu_\tau$ selection. **a** shows the number of reconstructed π^0 mesons. The dots are the data, the open histogram is the signal. The dark-shaded area represents the contribution from $\bar{K}^0 \pi^- \pi^0 \nu_\tau$ final states, the light-shaded area denotes other background channels. **b** is the kaon weight of the primary track

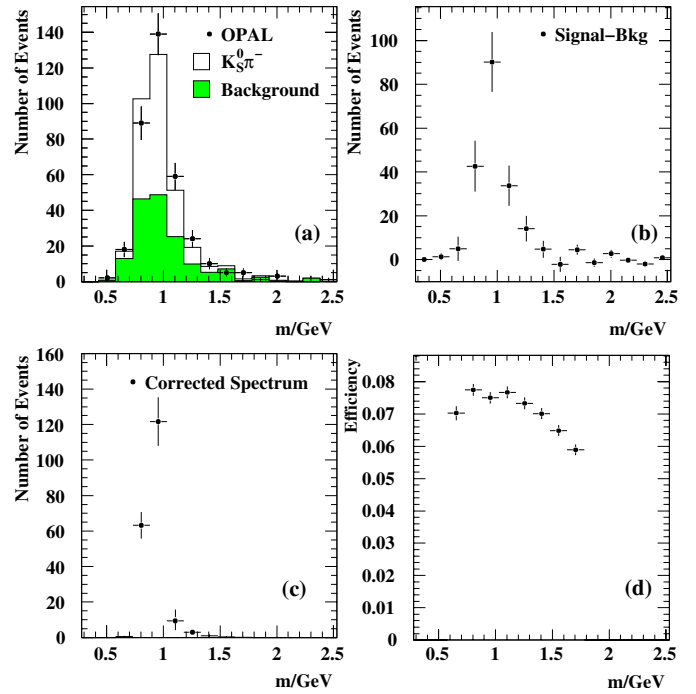


Fig. 11. Result of the $\bar{K}^0 \pi^- \nu_\tau$ selection. Plot **a** shows the measured invariant mass spectrum. The dots are the data, the open histogram is the Monte Carlo signal and the shaded area is the background. Plot **b** shows the background subtracted spectrum, **c** the corrected spectrum. Note that the correlation between neighboring bins in the corrected spectrum is of the order of 50%. Plot **d** is the selection efficiency as a function of the invariant mass

kaons as defined in Sect. 3.3. These two requirements reduce the background from photon conversions and decays containing K_S^0 .

To identify the kaon in the decay, one of the like-sign candidate tracks must have $p > 3$ GeV and $W_\pi < -0.9$. To further reduce the pion background among these candidate tracks, W_K and W_π are input to a neural network. The track is rejected if the output of the neural network is below 0.3 (see Fig. 12c). Exactly one like-sign track is allowed to fulfill these requirements, otherwise the decay is treated as background. If the momentum of the unlike-sign track is consistent with the $\tau^- \rightarrow \pi^- K^- K^+ \nu_\tau$ hypothesis, this τ decay is only accepted if $W_\pi > -0.95$ (see Fig. 12d).

The algorithm for identifying neutral pions (Sect. 3.2) is then applied to the selected cones. For this channel, the number of reconstructed π^0 mesons with an energy greater than 2 GeV is required to be zero (see Fig. 12b). Otherwise this τ decay is treated as background. To further improve the purity of the selection, the cosine of the decay angle in the rest frame of the τ lepton, the so-called Gottfried-Jackson angle Θ^* is calculated. The $\cos \Theta^*$ distribution is shown in Fig. 12a. For events where the kaon hypothesis was applied to the wrong track or the number of identified neutral pions does not correspond to the true number, this calculation leads to unphysical values of that variable. Due to resolution effects, correctly identified signal events can also give values beyond ± 1 . Therefore, a cut was applied at $\cos \Theta^* = \pm 1.2$. The contribution from

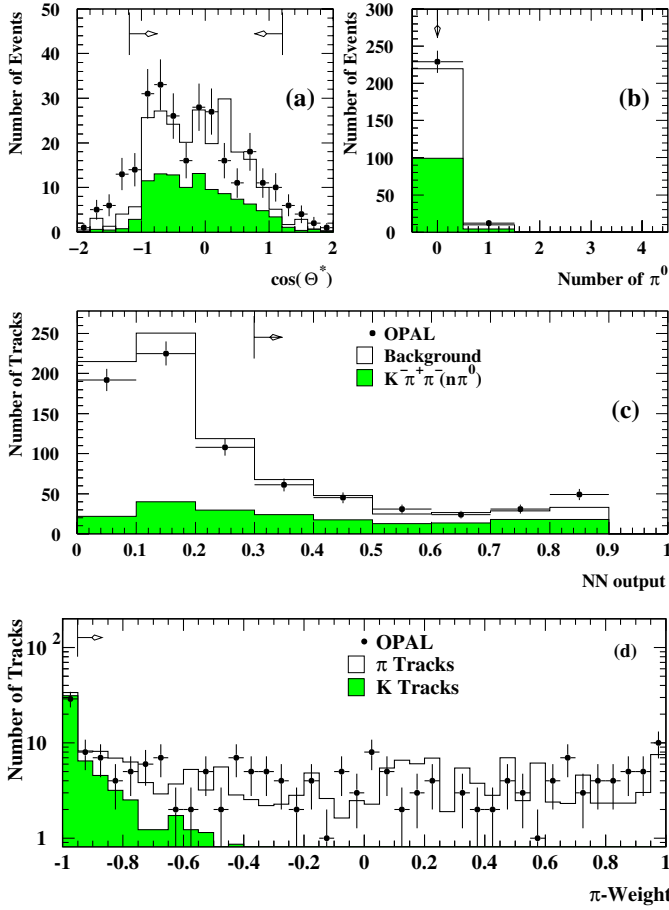


Fig. 12. Variables used in the $K^-\pi^+\pi^-(n\pi^0)\nu_\tau$ selection. Plot **a** shows the cosine of the Gottfried-Jackson angle Θ^* , **b** the number of reconstructed neutral pions and **c** the output of the neural network. The dots are the data, the open histogram is the background prediction from the Monte Carlo and the shaded histogram denotes the contribution from $K^-\pi^+\pi^-(n\pi^0)\nu_\tau$ final states. **d** shows the pion weight of the unlike-sign track in the $K^-\pi^+\pi^-(n\pi^0)\nu_\tau$ selection. Here the open histogram denotes the contribution from pion tracks, the shaded histogram is the contribution from kaon tracks. The dots are the data. For all plots, all selection cuts have been applied except for the cut on the variable shown. The arrows indicate the region selected

$\tau^- \rightarrow K^-\pi^+\pi^-(n\pi^0)\nu_\tau$ events is included in the background estimate.

From this selection 269 events are seen in the data with a contribution of 149.8 background events predicted from Monte Carlo. This corresponds to a background fraction of 63%. The main background contribution comes from decays $\tau^- \rightarrow \pi^-\pi^-\pi^+\nu_\tau$, $\tau^- \rightarrow \pi^-K^-K^+\nu_\tau$ and $\tau^- \rightarrow K^-\pi^+\pi^-\pi^0\nu_\tau$, where the π^0 meson escapes detection. The invariant mass spectrum can be found in Fig. 13. The mass resolution in this channel is approximately 20 MeV.

4.3.2 $\bar{K}^0\pi^-\pi^0\nu_\tau$

Exactly one identified K_S^0 and exactly one π^0 meson with an energy $E^{\pi^0} > 2$ GeV is required for this final state. The

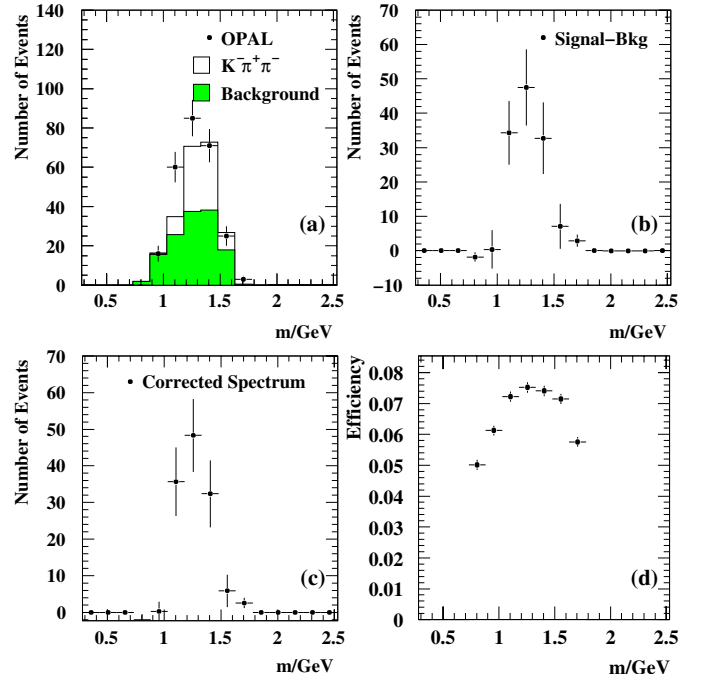


Fig. 13. Result of the $K^-\pi^+\pi^-\nu_\tau$ selection. Plot **a** shows the measured invariant mass spectrum. The dots are the data, the open histogram is the Monte Carlo signal and the shaded area is the background. Plot **b** shows the background subtracted spectrum, plot **c** is the corrected spectrum. Note that the correlation between neighboring bins in the corrected spectrum is of the order of 50%. Plot **d** is the selection efficiency as a function of the invariant mass

pion candidate track has to satisfy the same requirements as for $(K\pi)^-$ final states.

From this selection, 65 events are expected and 67 are seen in the data with a background fraction of 72%. The main background contribution comes from decays $\tau^- \rightarrow \bar{K}^0\pi^-\nu_\tau$ where a fake π^0 was reconstructed. The invariant mass spectrum can be seen in Fig. 14. The mass resolution in this channel is approximately 100 MeV.

4.4 The $(K\pi\pi\pi)^-\nu_\tau$ final state

The $(K\pi\pi\pi)^-$ signal consists of the following final states: $K^-\pi^+\pi^-\pi^0\nu_\tau$, $\bar{K}^0\pi^-\pi^0\pi^0\nu_\tau$, $K^-\pi^0\pi^0\pi^0\nu_\tau$ and $\bar{K}^0\pi^-\pi^+\pi^-\nu_\tau$. From these, only the first one which has the highest branching fraction is investigated. The same procedure as for the $K^-\pi^+\pi^-\nu_\tau$ channel is used. In addition, one identified π^0 meson with an energy of more than 2 GeV is required. The invariant mass spectrum can be seen in Fig. 15. The mass resolution in this channel is approximately 60 MeV. From this selection, 14 events are seen in the data with a contribution of 10 events from background. The selection efficiency is of the order of 1%. The main background contribution comes from $\tau^- \rightarrow K^-\pi^+\pi^-\nu_\tau$ decays, where one fake neutral pion was identified.

The number of signal events selected in the data for this final state is not significantly different from zero. Therefore,

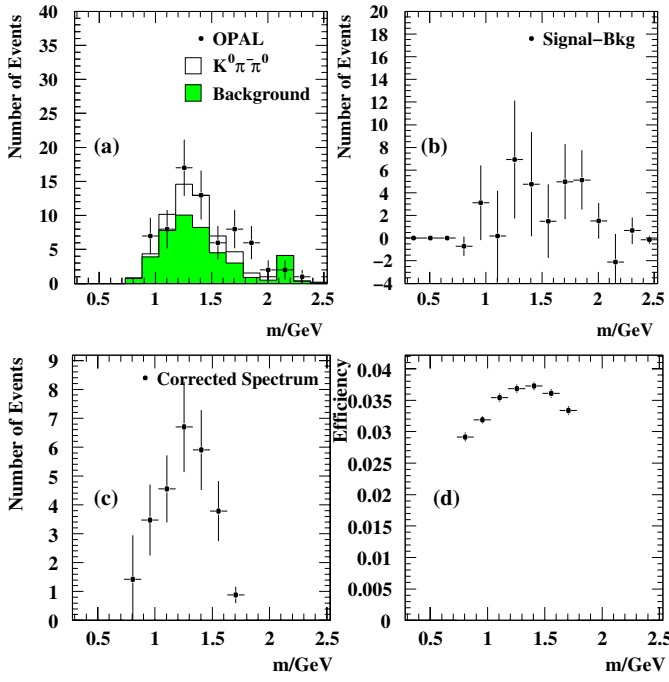


Fig. 14. Result of the $\bar{K}^0 \pi^0 \pi^0 \nu_\tau$ selection. Plot **a** shows the measured invariant mass spectrum. The dots are the data, the open histogram is the Monte Carlo signal and the shaded area is the background. Plot **b** shows the background subtracted spectrum, **c** the corrected spectrum. Note that the correlation between neighboring bins in the corrected spectrum is of the order of 50%. Plot **d** is the selection efficiency as a function of the invariant mass

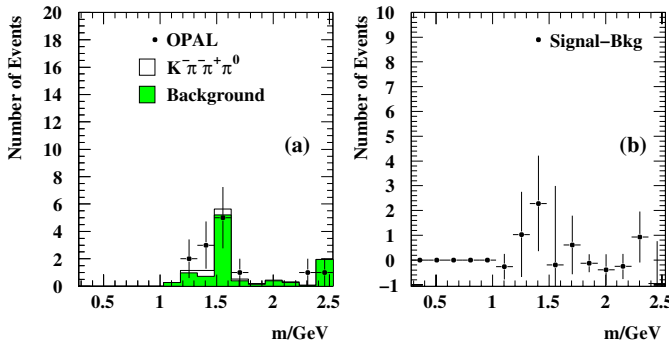


Fig. 15. Result of the $K^- \pi^+ \pi^- \pi^0 \nu_\tau$ selection. Plot **a** shows the measured invariant mass spectrum. The dots are the data, the open histogram is the Monte Carlo signal and the shaded area is the background. Plot **b** shows the background subtracted spectrum

like for the other four-meson final states with strangeness, the prediction from Monte Carlo is used throughout the rest of this paper.

5 Results

5.1 Branching fractions

The measured data used in the spectral function analysis allows the determination of competitive branching fractions

for the channels $\tau^- \rightarrow K^- \pi^0 \nu_\tau$ and $\tau^- \rightarrow K^- \pi^+ \pi^- \nu_\tau$. The expected number of events for these channels is calculated via

$$N_i = N_i^{\text{non-}\tau} + (1 - f_{\text{bkg}}^{\text{non-}\tau}) \cdot N^\tau \sum_j \varepsilon_{ij} B_j F_j^{\text{Bias}}.$$

Here, i is the signal channel under consideration and the index j runs over all τ decay channels. N_i is the number of decays observed in the data, i.e. the number of τ cones passing the cuts. The number of decays from the non- τ background is denoted by $N_i^{\text{non-}\tau}$, $f_{\text{bkg}}^{\text{non-}\tau}$ is the fraction of non- τ background, N^τ is the number of τ events, ε_{ij} the efficiency matrix, F_j^{Bias} the bias factor for the respective channel due to τ preselection cuts and B_j are the fitted branching fractions.

The branching fractions for the two signal channels mentioned above were allowed to vary freely. The branching fractions of all other τ decay channels with strangeness and the branching fraction of those channels contributing to the background prediction were allowed to vary only within their PDG errors [22]. The fit function reads

$$\chi^2 = \sum_{K^- \pi^+ \pi^- \nu_\tau}^{K^- \pi^0 \nu_\tau} \left(\frac{N_{\text{meas}} - N_{\text{exp}}}{\sigma} \right)^2 + \sum_{j \in \text{other}} \left(\frac{B_j - B_{j, \text{PDG}}}{\sigma_j} \right)^2,$$

where the sum runs over the final states which have been included in the respective terms. The selection quantities used in the fit together with the contributions from the background channels and their branching fractions used in the Monte Carlo simulation are shown in Table 2. From the fit, the following results are obtained:

$$\begin{aligned} B(\tau^- \rightarrow K^- \pi^0 \nu_\tau) &= (0.471 \pm 0.059_{\text{stat}} \pm 0.023_{\text{sys}}) \%, \\ B(\tau^- \rightarrow K^- \pi^+ \pi^- (0\pi^0, \text{ex.}\bar{K}^0) \nu_\tau) &= (0.415 \pm 0.053_{\text{stat}} \pm 0.040_{\text{sys}}) \%, \end{aligned}$$

where the possibility that the $\pi^+ \pi^-$ pair in the $K^- \pi^+ \pi^- \nu_\tau$ final state comes from a K_S^0 has been excluded. For the estimation of the systematic uncertainty, the following sources are considered. They are summarized in Table 3. The total systematic uncertainty is obtained by adding the individual contributions in quadrature.

- Branching fractions (Δ_B):
The error on the branching fractions for the background channels contributes to the systematic uncertainty. The fit is therefore repeated with these branching fractions fixed to their PDG values. The difference in the statistical error and the size of the shift in the central value as obtained from this fit, added in quadrature, is quoted as systematic uncertainty.
- Energy loss measurement ($\Delta_{dE/dx}$):
In the selection, the specific energy loss dE/dx is used

Table 2. Quantities used in the fit for the branching fractions. The errors quoted for efficiency, bias factor and background fractions are from Monte Carlo statistics only. The last column contains the branching fractions for the background channels used in the Monte Carlo simulation [22]

$\tau^- \rightarrow K^- \pi^0 \nu_\tau$			
No. of Events	360		
Selection Efficiency / %	8.42 ± 0.17		
Preselection Bias Factor	1.016 ± 0.011		
Non- τ Background Fraction	0.006 ± 0.004		
τ Background Fraction	0.540 ± 0.027		
	Bkg. Fraction	Efficiency / %	$B^{\text{PDG}}/\%$
$\pi^- \pi^0 \nu_\tau$	13.5%	0.051 ± 0.005	25.41 ± 0.14
$K^- K^0 \pi^0 \nu_\tau$	9.9%	6.0 ± 0.3	0.155 ± 0.020
$K^- \nu_\tau$	8.1%	1.25 ± 0.06	0.686 ± 0.023
$K^- K^0 \nu_\tau$	7.3%	4.5 ± 0.2	0.154 ± 0.016
$\pi^- \pi^0 \pi^0 \nu_\tau$	6.2%	0.07 ± 0.01	9.17 ± 0.14
$K^- \pi^0 \pi^0 \nu_\tau$	5.0%	9.6 ± 0.5	0.058 ± 0.023
$K^- \pi^0 \pi^0 \pi^0 \nu_\tau$	2.6%	8.9 ± 0.6	0.037 ± 0.021
other	1.4%		
$\tau^- \rightarrow K^- \pi^+ \pi^- \nu_\tau$			
No. of Events	269		
Selection Efficiency / %	6.59 ± 0.06		
Preselection Bias Factor	0.953 ± 0.013		
Non- τ Background Fraction	0.007 ± 0.006		
τ Background Fraction	0.631 ± 0.044		
	Bkg. Fraction	Efficiency / %	$B^{\text{PDG}}/\%$
$\pi^- \pi^+ \pi^- \nu_\tau$	21.6%	0.15 ± 0.02	9.22 ± 0.10
$\pi^- K^- K^+ \nu_\tau$	10.3%	3.9 ± 0.2	0.161 ± 0.019
$\pi^- \pi^+ \pi^- \pi^0 \nu_\tau$	8.1%	0.5 ± 0.1	4.24 ± 0.10
$K^- \pi^+ \pi^- \pi^0 \nu_\tau$	6.7%	2.7 ± 0.2	0.064 ± 0.024
other	16.4%		

Table 3. Individual contributions to the systematic uncertainty of the branching fraction measurements as explained in the text

	$\tau^- \rightarrow K^- \pi^0 \nu_\tau$	$\tau^- \rightarrow K^- \pi^+ \pi^- \nu_\tau$
Branching Fractions	0.005	0.026
Energy Loss Measurement	0.012	0.019
Energy Scale	0.010	0.011
Momentum Scale	0.003	0.003
MC Statistics	0.014	0.021
Bias Factor	0.004	0.005
Total	0.023	0.040

to separate pions from kaons. Cuts on corresponding weights are applied which are calculated from the pull distribution (see Fig. 3). A possible shift in this quantity can lead to a systematic misidentification of tracks. The pull distribution is therefore shifted within the error on its mean and the selection procedure is repeated. The difference between the branching fractions obtained with and without the shift applied is the systematic uncertainty.

– Energy scale in π^0 reconstruction (Δ_E):
The energy resolution can be tested by measuring the invariant two-photon mass from π^0 decays. A systematic shift in the observed mass in the data compared to the detector simulation can be translated into a scale factor for the reconstructed photon energies. Deviations of (0.5 ± 0.9) MeV from the nominal π^0 mass have been observed [7], corresponding to a scale factor of 1.004 ± 0.007 . The energies of the reconstructed pho-

tions in the Monte Carlo samples were therefore varied by $\pm 0.7\%$. The difference between the branching fractions obtained with and without the variation applied is the systematic uncertainty.

– Momentum scale (Δ_p):

The systematic uncertainty connected with the momentum scale was tested using $Z^0 \rightarrow \mu^- \mu^+$ events [7]. The difference in momentum resolution between data and Monte Carlo as a function of $\cos \theta$ was studied. To assess the systematic uncertainty Δ_p in hadronic τ decays, all particle momenta in the Monte Carlo were varied accordingly. The difference in the result with and without this variation is quoted as a systematic uncertainty.

– Monte Carlo statistics (Δ_{MC}):

The precision of the background prediction depends on the Monte Carlo statistics used in the selection procedure. Therefore, the number of background events selected is varied randomly within its statistical uncertainty. The observed spread in the branching fraction due to this variation is quoted as a systematic uncertainty.

– Bias factor (Δ_{FBias}):

The bias factors determined from the Monte Carlo are varied by their uncertainty and the branching fractions are then refitted. The observed spread due to this variation contributes to the total systematic uncertainty.

5.2 Improved averages for $B(\tau^- \rightarrow K^- \pi^0 \nu_\tau)$ and $B(\tau^- \rightarrow K^- \pi^+ \pi^- \nu_\tau)$

For the determination of the spectral function and its moments described below, new average values for the branching fractions of the decays $\tau^- \rightarrow K^- \pi^0 \nu_\tau$ and $\tau^- \rightarrow K^- \pi^+ \pi^- \nu_\tau$ are determined. In addition to the results obtained here, the same measurements are used as inputs for the calculation as in [22]. For the channel $\tau^- \rightarrow K^- \pi^+ \pi^- \nu_\tau$, the previous result from OPAL [40] was replaced by the measurement obtained in this analysis. The CLEO result was replaced using [41]. The new averages are:

$$B_{av}(\tau^- \rightarrow K^- \pi^0 \nu_\tau) = (0.453 \pm 0.030)\%$$

$$B_{av}(\tau^- \rightarrow K^- \pi^+ \pi^- \nu_\tau) = (0.330 \pm 0.028)\%.$$

The measurements used, together with the averages given in [22] and the improved values for the branching fractions obtained here are displayed in Fig. 16.

The branching fraction obtained from this analysis for the $K^- \pi^0 \nu_\tau$ channel is consistent with the previous measurements within the errors quoted. The value obtained for the $K^- \pi^+ \pi^- \nu_\tau$ channel is consistent with the new measurement from CLEO and the theoretical prediction in [30]. The ALEPH [42] result differs from these values by roughly 2.5σ .

5.3 The strangeness spectral function

The hadronic decay of the τ lepton is commonly written in terms of the so-called spectral functions $v_1(s)$, $a_{0/1}(s)$

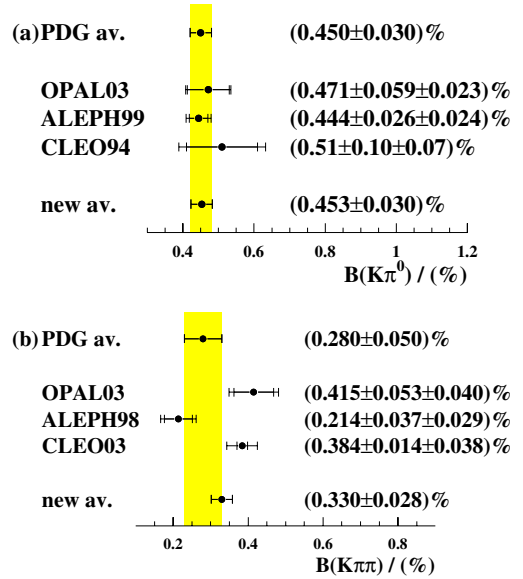


Fig. 16. New averages for the branching fractions for $\tau^- \rightarrow K^- \pi^0 \nu_\tau$ **a** and $\tau^- \rightarrow K^- \pi^+ \pi^- \nu_\tau$ **b**. The previous world average as given by the PDG is indicated by the shaded band. The measurements used in the calculation of the weighted mean and the new averages are shown. OPAL03 denotes the values for the branching fractions obtained from this analysis

for the non-strange part and $v_{0/1}^S(s)$ and $a_{0/1}^S(s)$ for the strange part. The functions v and a are the vector (V) and the axialvector (A) contributions, respectively, while the subscript denotes the angular momentum J . The variable s is the invariant mass squared of the hadronic system. The spectral function is experimentally determined by measuring the invariant mass spectra of the given hadronic modes and normalizing them to their respective branching fractions. The contributions to the total strangeness spectral function then read:

$$\begin{aligned} & v_1^S(s)/a_1^S(s) \\ &= \frac{m_\tau^2}{6|V_{us}|^2 S_{ew}} \left(1 - \frac{s}{m_\tau^2}\right)^{-2} \left(1 + \frac{2s}{m_\tau^2}\right)^{-1} \\ & \times \frac{B(\tau \rightarrow (V/A)^{(S=-1, J=1)} \nu_\tau)}{B(\tau \rightarrow e^- \bar{\nu}_e \nu_\tau)} \frac{1}{N_{V/A}} \frac{dN_{V/A}}{ds} \end{aligned} \quad (3)$$

and

$$\begin{aligned} & v_0^S(s)/a_0^S(s) \\ &= \frac{m_\tau^2}{6|V_{us}|^2 S_{ew}} \left(1 - \frac{s}{m_\tau^2}\right)^{-2} \\ & \times \frac{B(\tau \rightarrow (V/A)^{(S=-1, J=0)} \nu_\tau)}{B(\tau \rightarrow e^- \bar{\nu}_e \nu_\tau)} \frac{1}{N_{V/A}} \frac{dN_{V/A}}{ds} \end{aligned} \quad (4)$$

where $|V_{us}| = 0.2196 \pm 0.0023$ [22] is the CKM weak mixing matrix element, $m_\tau = (1776.9^{+0.31}_{-0.27})$ MeV [43] and $S_{ew} = 1.0194 \pm 0.0040$ [44] is an electroweak correction factor. The total strangeness spectral function ($v + a$) is

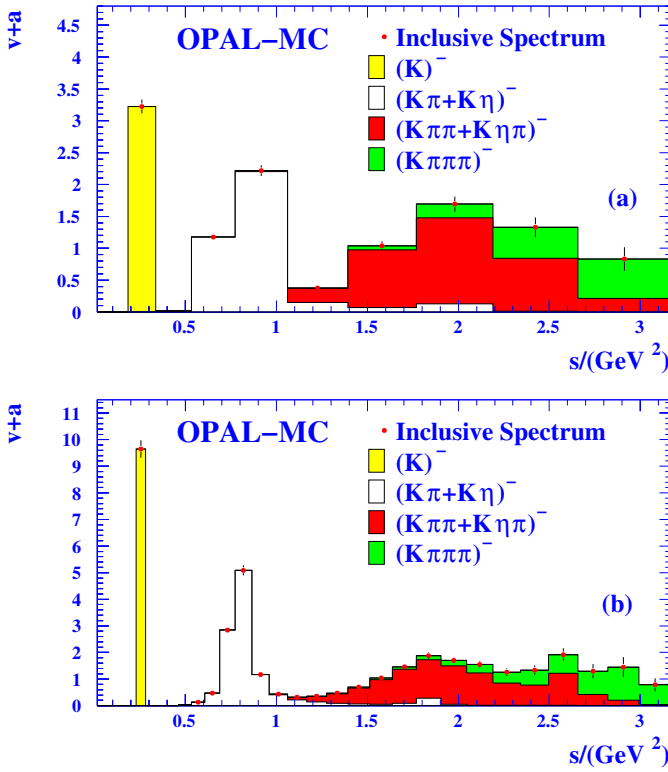


Fig. 17. The strangeness spectral function as determined from Monte Carlo. For illustration purposes two different binnings were used – **a** 150 MeV and **b** 50 MeV in mass. The dots represent the inclusive spectrum. The white histogram denotes the contribution from the $(K\pi)^-$ and $(K\eta)^-$ final states, the dark shaded histogram those from the $(K\pi\pi)^-$ and $(K^*(892)\eta)^-$ channels and the light shaded area shows the contribution from $(K\pi\pi\pi)^-$ final states. The leftmost bin represents the kaon pole which is the only contribution to the spin-0 part. The errors shown include the statistical uncertainty of the Monte Carlo as well as the uncertainties on the branching fractions

then obtained by adding the individual contributions. To disentangle the vector and the axialvector contributions for the spin-1 part, a detailed analysis of the resonance structure of the measured spectra is necessary which is not done here due to the limited statistics. The kaon pole contributes to the pseudoscalar spin-0 part.

Summing the individual contributions (see Equations 3 and 4), the strangeness spectral function is obtained. The Monte Carlo prediction of the total strangeness spectral function as a function of the invariant mass squared is displayed in Fig. 17. The improved version of the τ Monte Carlo as explained in Sect. 2.3 has been used here. The branching fractions as given in Table 1 have been used to weight the individual mass spectra. For illustration purposes the spectral function is shown using two different binnings. A non-equidistant binning is chosen which corresponds to a bin width of 50 MeV and 150 MeV in the invariant mass, respectively. The errors given here are from Monte Carlo statistics only.

The spectral function obtained from the data is displayed in Fig. 18. For $\tau^- \rightarrow K^- \pi^0 \nu_\tau$ and $\tau^- \rightarrow K^- \pi^+ \pi^- \nu_\tau$,

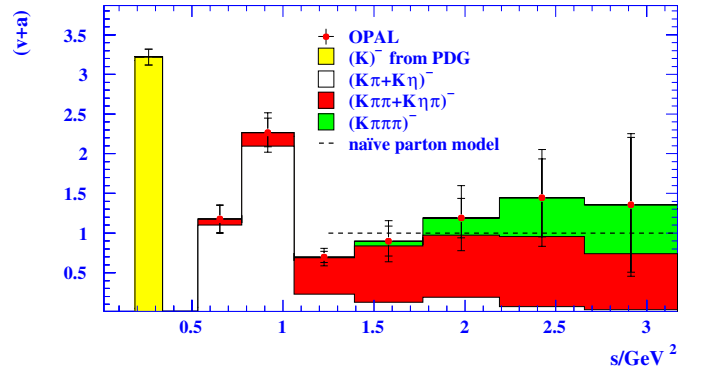


Fig. 18. The spectral function from strange τ lepton decays. The dots show the inclusive spectrum as measured in OPAL. The white histogram shows the exclusive $(K\pi+K\eta)^-$ spectrum, the dark shaded area the $(K\pi\pi+K\eta\pi)^-$ contribution and the light shaded area the contribution from the $(K\pi\pi\pi)^-$ final states. The error bars show the statistical uncertainty (inner error bar) and the systematic uncertainty added in quadrature (total error)

the new average branching fractions and their respective errors as given in Sect. 5.2 are used. The binning chosen in this plot corresponds to a bin width of 150 MeV in the invariant mass and is governed by the mass resolution of the $\bar{K}^0 \pi^- \pi^0 \nu_\tau$ final state. The dots with error bars represent the inclusive spectrum. The inner error bars are the statistical uncertainties. They include the uncertainty on the efficiency and on Monte Carlo statistics. The total error is calculated by adding up the statistical and systematic uncertainties (explained in the next section) in quadrature. The numerical values are given in Table 4. The systematic uncertainty is dominated by the uncertainty on the τ branching fractions.

For the $(K\pi)^-$ final state both τ decay channels $\bar{K}^0 \pi^- \nu_\tau$ and $K^- \pi^0 \nu_\tau$ are measured. The channel $K^- \eta \nu_\tau$ which also contributes to the two-meson final state is taken from Monte Carlo. For the $(K\pi\pi)^-$ final state the spectra $\bar{K}^0 \pi^- \pi^0 \nu_\tau$ and $K^- \pi^+ \pi^- \nu_\tau$ are measured. The contribution from the decay $K^- \pi^0 \pi^0 \nu_\tau$ is added from Monte Carlo as well as the $K^*(892)^- \eta \nu_\tau$ channel which also contributes to the three-meson final states. For the $(K\pi\pi\pi)^-$ spectrum, which consists of the channels $K^- \pi^+ \pi^- \pi^0 \nu_\tau$, $\bar{K}^0 \pi^- \pi^0 \pi^0 \nu_\tau$, $K^- \pi^0 \pi^0 \pi^0 \nu_\tau$ and $\bar{K}^0 \pi^- \pi^+ \pi^- \nu_\tau$, the prediction from the Monte Carlo is taken.

5.4 Systematic uncertainties on the spectral function

The sources for possible systematic uncertainties listed below have been considered. Since the individual contributions are different for the different final states, the error is given for each bin in s separately in Table 4.

- Energy loss measurement ($\Delta_{dE/dx}$):

The systematic variation is described in Sect. 5.1. In addition, the momentum dependence of this variation has been investigated. No significant influence has been found.

Table 4. Result for the strangeness spectral function. The table shows the values of the total strangeness spectral function ($v + a$) and the statistical and the total systematic uncertainties for every bin in s . The individual contributions to the systematic uncertainty are discussed in the text. The total uncertainty quoted is the quadratic sum of the statistical and systematic uncertainties

$(s\text{-range})/\text{GeV}^2$	Δ_B	$\Delta_{dE/dx}$	$\Delta_{K_S^0}$	Δ_E	Δ_p	Δ_{mcorr}	$\Delta_{\text{sys}}^{\text{tot}}$	Δ_{stat}	$v + a$
(0.53, 0.77)	0.04	0.006	0.006	0.007	0.003	0.06	0.07	0.17	1.17 ± 0.18
(0.77, 1.06)	0.13	0.011	0.011	0.014	0.001	0.11	0.17	0.18	2.27 ± 0.25
(1.06, 1.39)	0.08	0.003	0.003	0.004	0.001	0.03	0.09	0.07	0.69 ± 0.11
(1.39, 1.77)	0.18	0.005	0.005	0.005	0.002	0.05	0.18	0.19	0.90 ± 0.26
(1.77, 2.19)	0.32	0.006	0.007	0.007	0.003	0.06	0.33	0.25	1.22 ± 0.41
(2.19, 2.66)	0.35	0.007	0.009	0.009	0.003	0.07	0.36	0.49	1.44 ± 0.61
(2.66, 3.17)	0.30	0.007	0.008	0.008	0.003	0.07	0.31	0.85	1.35 ± 0.90

- Energy scale in π^0 reconstruction (Δ_E):
The systematic variation is described in Sect. 5.1.
- Momentum scale (Δ_p):
The systematic variation is described in Sect. 5.1.
- PDG errors on branching fractions (Δ_B):
The dominant contribution to the systematic uncertainty comes from the uncertainty in the τ branching fractions (see Table 1). The branching fractions are varied randomly and the difference in the result is quoted as systematic uncertainty. The channels which populate the region of high s have branching fractions with relative errors close to 100% leading to large uncertainty in the spectral function itself. This also covers the uncertainties on the shape.
- K_S^0 identification ($\Delta_{K_S^0}$):
A possible origin for systematic effects in the K_S^0 identification is the estimation of the background using Monte Carlo. In particular the number of photon conversions found in τ lepton decays is not perfectly modeled. The cut on the χ^2 probability of the 2C constrained fit to K_S^0 (see Sect. 3.3) has been varied from 10^{-5} to 0.01 to estimate a possible systematic effect. This cut reduces the number photon conversions in the sample by one order of magnitude.
- Mass correction procedure (Δ_{mcorr}):
In order to assess the systematic uncertainty associated with the mass correction procedure, two possible sources have to be considered. The effect due to different input mass spectra and the effect due to the choice of the number of iterations. They have been studied using high statistics event samples. The systematic deviations using either flat, phase space, or resonance shaped input spectra (with an optimized number of iterations for each scenario) are about 5% in each mass bin. The deviations are largest when a flat input distribution is assumed. Because the true mass spectrum is certainly not flat, we consider 5% to be a conservative estimate for Δ_{mcorr} . The effect of the number of iterations was estimated using a phase space distribution for input spectrum and performing one additional iteration step. The change was found to be negligible compared to the effect of varying the input distribution.

5.5 The spectral moments $R_{\tau,S}^{kl}$

The moments of the spectral function calculated here are defined as:

$$\begin{aligned}
 R_{\tau,S}^{kl}(m_\tau^2) &= \int_0^{m_\tau^2} ds \left(1 - \frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \\
 &\times \sum_{\tau^- \rightarrow \nu_\tau X_S^-} \frac{B(\tau \rightarrow (V/A)^{(S=-1, J=0/1)} \nu_\tau)}{B(\tau \rightarrow e^- \bar{\nu}_e \nu_\tau)} \frac{dN_{V/A}}{N_{V/A} ds},
 \end{aligned} \tag{5}$$

where the sum runs over all final states with open strangeness. The values measured for the moments $kl = (00, 10, 11, 12, 13, 20, 21, 30, 40)$, their statistical and systematic uncertainties are given in Table 5. The various sources contributing to the systematic uncertainty have been discussed in Sect. 5.4.

The value for $R_{\tau,S}^{00}$ is calculated from the branching fractions only and is therefore independent of the measured spectra. The result obtained in this analysis (see Table 5) differs from the updated ALEPH result [21] ($R_{\tau,S}^{00} = 0.1625 \pm 0.0066$) by about one standard deviation. This is mostly due to the improved average in the $K^- \pi^+ \pi^- \nu_\tau$ branching fraction which is significantly higher than the one previously used. In addition, the world averages of the branching fraction for other τ decay modes like $\bar{K}^0 \pi^- \pi^0 \nu_\tau$ and $\bar{K}^0 \pi^- \nu_\tau$ are different from those used in the ALEPH analysis. The total strange branching fraction used in [16] was $B_{\text{strange}}^{\text{ALEPH}} = (2.865 \pm 0.118)\%$, whereas in this analysis, using improved averages, a result of $B_{\text{strange}} = (2.993 \pm 0.090)\%$ was obtained.

In addition to the spectral moments as given in Equation 5, the CKM-weighted difference of the corresponding strange and non-strange moments is given:

$$\delta R_\tau^{kl} = \frac{R_{\tau,\text{non-S}}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2}, \tag{6}$$

(see Table 5) where $R_{\tau,S}^{kl}$ are the strange moments and $R_{\tau,\text{non-S}}^{kl}$ is the sum of the vector- and axialvector non-

Table 5. The spectral moments for $kl = (00, 10, 11, 12, 13, 20, 21, 30, 40)$. The tables include the values for the non-strange moments, the strange moments, the weighted difference and the quotient of strange to non-strange moments. The values for the non-strange moments given are calculated from [7] using updated branching fractions. For the strange moments, statistical and systematic uncertainties are listed separately for the individual sources. The statistical uncertainty also contains the uncertainty on the branching fractions. The uncertainty on R_τ^{00} is the total error from the branching fractions as given in [22]. On the right hand side of each table, the correlations are given in percent. For the weighted differences δR_τ^{kl} , the experimental uncertainty and the uncertainty on the matrix element $|V_{us}|$ are given separately

kl	$R_{\tau,\text{non-S}}^{kl}$	00	10	11	12	13	20	21	30	40
00	3.469 ± 0.014	100								
10	2.493 ± 0.013	66	100							
11	0.549 ± 0.004	68	65	100						
12	0.203 ± 0.002	51	9	74	100					
13	0.092 ± 0.002	33	-26	33	86	100				
20	1.944 ± 0.011	55	93	45	-11	-40	100			
21	0.346 ± 0.003	59	86	88	35	-13	71	100		
30	1.597 ± 0.009	48	85	28	-24	-44	93	58	100	
40	1.362 ± 0.008	42	77	14	-30	-43	87	43	92	100

kl	$R_{\tau,S}^{kl}$	Δ_{stat}	$\Delta_{dE/dx}$	$\Delta_{K_S^0}$	Δ_E	Δ_P	Δ_{mcorr}	00	10	11	12	13	20	21	30	40
00	0.1677 ± 0.0050	0.0050	-	-	-	-	-	100								
10	0.1161 ± 0.0038	0.0035	0.0006	0.0006	0.0005	0.0002	0.0011	89	100							
11	0.0298 ± 0.0012	0.0011	0.0001	0.0001	0.0001	0.0001	0.0004	97	83	100						
12	0.0107 ± 0.0006	0.0005	0.0002	0.0002	0.0002	0.0001	0.0002	86	54	91	100					
13	0.0048 ± 0.0004	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001	74	36	78	97	100				
20	0.0862 ± 0.0028	0.0025	0.0006	0.0006	0.0006	0.0002	0.0008	75	97	66	32	13	100			
21	0.0191 ± 0.0007	0.0006	0.0001	0.0001	0.0001	0.0001	0.0002	92	96	92	66	47	87	100		
30	0.0671 ± 0.0022	0.0020	0.0005	0.0005	0.0004	0.0002	0.0006	66	92	54	19	1	99	78	100	
40	0.0539 ± 0.0018	0.0016	0.0003	0.0003	0.0003	0.0001	0.0005	60	87	46	11	-4	96	70	99	100

kl	δR_τ^{kl}	Δ_{exp}	$\Delta_{ V_{us} }$	00	10	11	12	13	20	21	30	40
00	0.184 ± 0.128	0.105	0.073	100								
10	0.224 ± 0.095	0.080	0.050	79	100							
11	-0.039 ± 0.028	0.025	0.013	88	67	100						
12	-0.008 ± 0.014	0.013	0.005	67	37	65	100					
13	-0.002 ± 0.009	0.009	0.000	49	20	48	53	100				
20	0.264 ± 0.070	0.059	0.037	64	74	51	20	5	100			
21	-0.031 ± 0.017	0.015	0.008	81	76	75	46	26	66	100		
30	0.294 ± 0.055	0.047	0.029	56	71	42	11	0	73	60	100	
40	0.320 ± 0.045	0.038	0.023	52	69	36	6	-4	74	55	77	100

kl	$R_{\tau,S}^{kl}/R_{\tau,\text{non-S}}^{kl}$	00	10	11	12	13	20	21	30	40
00	0.0484 ± 0.0015	100								
10	0.0466 ± 0.0015	79	100							
11	0.0543 ± 0.0022	88	67	100						
12	0.0527 ± 0.0030	67	37	66	100					
13	0.0518 ± 0.0045	49	20	48	53	100				
20	0.0444 ± 0.0015	64	74	51	20	5	100			
21	0.0552 ± 0.0021	80	76	75	46	26	66	100		
30	0.0420 ± 0.0014	57	71	42	12	0	73	59	100	
40	0.0400 ± 0.0013	53	69	36	7	-4	73	55	76	100

Table 6. Comparison of the results for the weighted difference $\delta R_{\tau,S}$ of the strange and non-strange spectral moments as obtained by the OPAL (left) and the ALEPH (right) collaboration. Here, a unitarity constraint has been applied to the elements of the weak mixing matrix before the calculation. The experimental uncertainty quoted includes the statistical and the systematic uncertainties added in quadrature. The uncertainty due to V_{us} is given separately

kl	OPAL			ALEPH		
	$\delta R_{\tau,S}$	Δ_{exp}	$\Delta_{ V_{us} }$	$\delta R_{\tau,S}$	Δ_{exp}	$\Delta_{ V_{us} }$
00	0.262±0.117	0.102	0.058	0.374±0.133	0.118	0.062
10	0.278±0.088	0.078	0.040	0.398±0.077	0.065	0.042
20	0.304±0.065	0.058	0.030	0.399±0.054	0.044	0.031
30	0.325±0.051	0.046	0.023	0.396±0.042	0.034	0.024
40	0.344±0.042	0.037	0.019	0.395±0.034	0.028	0.020

strange moments from [7], using updated branching fractions. The values of the matrix elements used for the non-strange and strange moments are $|V_{ud}| = (0.9734 \pm 0.0008)$ and $|V_{us}| = (0.2196 \pm 0.0023)$, respectively [22]. The experimental uncertainty and the uncertainty due to the CKM matrix element are quoted separately. Systematic errors which are common to both, strange and non-strange moments, such as the energy scale error and the momentum scale for tracks, are 100% correlated and are treated accordingly.

A comparison of the values for the weighted differences obtained in this work with previous ALEPH results [16] displays some differences in the central values. This is partly due to the updated branching fractions of strange and non-strange τ decay channels. Since there is a strong cancellation between the weighted strange and non-strange moments, the result is particularly sensitive to the matrix elements used as weighting factors. Small differences in the strange spectral moments therefore can lead to large deviations in the weighted differences as observed here. The ALEPH values were obtained using unitarity constrained matrix elements. In order to be able to compare the results from the two experiments, the OPAL values have been re-evaluated using constrained matrix elements. They are shown together with the ALEPH values in Table 6, where the experimental uncertainty and the uncertainty due to the matrix element $|V_{us}|$ are given separately. The remaining difference for the moments $k \neq 0$ is then due to the differences in the shape of the strangeness spectral function.

6 Conclusions

The spectral function in τ lepton decays into vector and axial vector final states with open strangeness, as well as the spectral moments $R_{\tau,S}^{kl}$, have been measured with the OPAL detector, operating on the Z^0 resonance. The selection of the decay channels requires a detailed study of the charged kaon identification using dE/dx , neutral pion reconstruction in the electromagnetic calorimeter, and identification of K_S^0 . For the decays $\tau^- \rightarrow (K\pi)^- \nu_\tau$, $(K\pi\pi)^- \nu_\tau$ and $(K\pi\pi\pi)^- \nu_\tau$,

93.4% of all final states have been experimentally accessed; for the remaining 6.6%, as well as for the strange final states including η mesons, Monte Carlo simulations have been used. The reconstructed strange final states corrected for resolution effects and detection efficiencies yield the strangeness spectral function of the τ lepton. The spectral moments $R_{\tau,S}^{kl}$ for $kl = \{10, 11, 12, 13, 20, 21, 30, 40\}$, as well as the ratio of strange to non-strange moments, updated from [7], and the weighted differences δR_{τ}^{kl} are given, which are useful quantities for further theoretical analyses. For the decay channels $\tau^- \rightarrow K^- \pi^0 \nu_\tau$ and $\tau^- \rightarrow K^- \pi^+ \pi^- \nu_\tau$ competitive branching fractions are obtained:

$$B(\tau^- \rightarrow K^- \pi^0 \nu_\tau) = (0.471 \pm 0.059_{\text{stat}} \pm 0.023_{\text{sys}})\%$$

$$B(\tau^- \rightarrow K^- \pi^+ \pi^- \nu_\tau) = (0.415 \pm 0.053_{\text{stat}} \pm 0.040_{\text{sys}})\%.$$

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